A DETERMINISTIC MODEL FOR PREDICTING WATER YIELD FROM TWO DIFFERENT WATERSHEDS

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ABSTRACT

This study was conducted in Pogung-Code sub watershed and Pulo-Opak sub watershed from 1993 to 2002. A deterministic model developed by Haan (1972) was used to determine the amount of water yield from watersheds. The model consists of four parameters, namely: (i) maximum rate of seepage from the soil water zone in mm per day (S), (ii) maximum soil moisture less readily available for evapotranspiration in mm (C), (iii) fraction of seepage from the soil water zone that becomes runoff (F), and (iv) maximum infiltration rate in mm per hour (f_{max}).

The inputs required by the model were the daily rainfall and the estimated potential evapotranspiration which was computed using Penman method. Streamflow records for 10 years of Pogung-Code sub watershed (2,801.77 ha) and Pulo-Opak sub watershed (4,856.37 ha) were used to test the validity of the model. The parameters obtained for Pogung-Code sub watershed were: S = 0.70 mm per day, C = 112.32 mm, F = 0.63, and f_{max} = 5.38 mm per hour, meanwhile for Pulo-Opak sub watershed were: S = 2.33 mm per day, C = 86.72 mm, F = 0.26 and f_{max} = 5.68 mm per hour.

The final test of the adequacy of the model lay in a comparison of observed and simulated runoff. The comparison showed that the observed and simulated runoff values are not significantly different. This was based on the results obtained from statistical measures to test the model. The model did a better simulation in the smaller watershed (Pogung-Code sub watershed) than in the larger one (Pulo-Opak sub watershed).

Key words: deterministic model, watershed, water yield, soil water zone.

INTRODUCTION

Many studies have been directed at correlating runoff with rainfall, agricultural practices, land cover conditions, and the physical characteristics of the watershed (Leavesly and Stannard in Singh, 1995; Zhao, 1993; Spruill, et al., 2000; Yuan, et al., 2001). Generally, these studies result in prediction equations that are useful for estimating annual water yields but are restricted to certain regions and to a limited range of watershed characteristics.

A few comprehensive watershed models have been developed that are capable of synthesizing continuous streamflow records. One of the most successful of these was developed at Stanford University and its known as the Stanford Watershed Model (Chow, 1988). This model has been used in many investigation with various degrees of success. Its major drawback is the time required for computer processing and difficulty in estimating the many parameters required.

This study attempts to construct such a model based on the work of Haan (1972), using easily measurable input parameters, to predict monthly runoff from small watersheds. The model was tested using data from Pogung and Pulo sub watersheds, where daily rainfall, streamflow and other climatic data have been recorded properly.

Objectives of the study

This study aims:

1. to predict monthly streamflow from daily rainfall data,
2. to estimate the parameters of the model that illustrate the characteristics of the watersheds.

Theoretical Framework

All runoff has precipitation as its primary source. Precipitation reaches the stream channel by four primary routes, namely: (1) channel interception, which is precipitation that falls directly into the water in the stream channels; (2) surface runoff, which is the portion of precipitation that does not infiltrate into the soil but flows over the surface until it reaches a channel; (3) subsurface flow or interflow, which comes from precipitation that infiltrates into the soil but is impeded in its downward course by a layer of restricted permeability. Parts of the precipitation that enter the soil may be retained by molecular forces on the surface of the soil particles or in the soil pores. Only that portion of the precipitation entering the soil that exceeds the soil moisture storage capacity and does not percolate to deeper levels becomes subsurface flow; and (4) base flow, which is the portion of precipitation that percolates deep into the soil and is released slowly, sustaining streamflow during long, dry periods (Haan, 1972).

There is no strict division between subsurface flow and base flow. It is, however, convenient to make one (Satterlund, 1972). The term of subsurface flow is used to indicate the flow through the upper mantle, which is usually identifiable as resulting from a single storm. When there is one storm quickly follows another, the distinction between surface flow and base flow may become blurred.

In a runoff prediction model, the laws of conservation of mass are included in the set of theoretical principles used to explain the hydrologic cycle (Woolhiser, 1982).

An equation expressing the integral of the conservation of mass for some arbitrary time interval (Δt) can be written as :

water input = water output $\pm$ change of water storage.

That is,

$$P = Q_S + Q_R + E \pm \Delta S \qquad (1)$$

where:

- P = precipitation received in area A
- Q_s = net surface runoff
- Q_R = net subsurface flow
- E = evapotranspiration per unit area
- ΔS = change in soil water storage

METHODOLOGY

The study is aimed at applying a deterministic and lumped hydrologic model capable of simulating monthly streamflow from two different watersheds. Constraints placed on the model included simplicity, ease of application, and minimum input requirements. Furthermore, it is to have a minimum number of parameters, which could be easily estimated from short period of observed streamflow records. This portion of the model was a modification of early works done by Haan (1972, 1976) and Jarboe and Haan (1974). A schematic representation of the model is given in Figure 1.

The moisture holding capacity of the soil is divided into volume M_R, which is soil moisture readily available for evapotranspiration, and volume M_L, which is soil moisture less readily available for evapotranspiration. The maximum capacity of M_R is 25 mm of water. The 25 mm value for soil moisture utilized as the potential rate was justified as being the approximate amount of moisture held in the upper foot of soil between field capacity and one atmosphere of tension (Denmead and Shaw, 1962; Ligon, et al., 1965). The maximum capacity of M_L is C.

Evapotranspiration is possibly the most important determinant of water yield. The model merely assumes that average conditions exist for producing evapotranspiration unless rain occurs. Estimated daily evapotranspiration (E) is potential evapotranspiration (E_p) reduced by factors for soil dryness (Ligon et al., 1965). On days without rainfall, evapotranspiration is equal to potential evapotranspiration as long as soil water is readily available and then is reduced by the ratio of M_L to C.

On days when measurable rainfall (P_d) occurs, the evapotranspiration rate is taken as one-half of potential value (Haan, 1972). This practice partially compensates the higher humidity, lower temperatures, and decreased solar radiation usually associated with rainy days.

The daily evapotranspiration (E) was determined from :

$$E = E_p; \qquad (P_d = 0;\ \text{and}\ 0 < M_R \le 25) \tag{2}$$

$$E = E_p\left(\frac{M_L}{C}\right); \quad (Pd = 0, \text{and}\ MR = 0) \tag{3}$$

$$E = \frac{E_P}{2} \qquad (P_d \ge 0{,}25\ \text{and}\ M_R \le 25) \tag{4}$$

$$E = \frac{E_P}{2}\left(\frac{M_L}{C}\right); \quad (P_d > 0{,}25\ \text{and}\ M_R = 0) \tag{5}$$

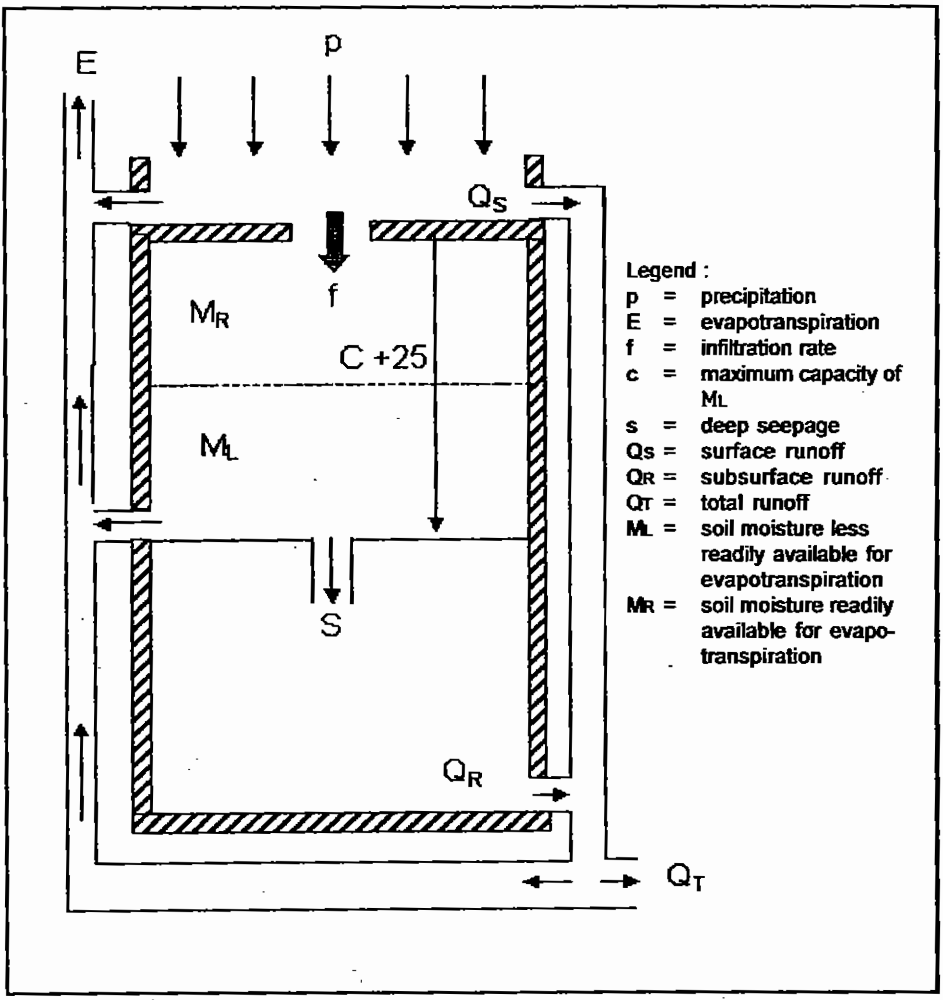

Figure 1. Mathematical representation of the model (After Ha an, 1972)

where :

E_p = potential daily evapotranspiration (mm),

P_d = depth of rainfall that occurred on the day in question (mm),

M_R = soil moisture readily available for evapotranspiration having a maximum capacity of 25 mm of water,

M_L = soil moisture less readily available for evapotranspiration having a maximum capacity of C.

The infiltration rate (f) was determined from

$$f = f_{max} \quad \text{for } P > f_{max}, \; M_R < 25 \text{ or } M_L < C \tag{6}$$

$$f = P \quad \text{for } P \leq f_{max}, \; M_R < 25 \text{ or } M_L < C \tag{7}$$

$$f = 0 \quad \text{for } M_R = 25 \text{ and } M_L = C \tag{8}$$

where f_{max} is the maximum possible infiltration rate and P is the precipitation rate. What is termed infiltration by the model may in fact be a combination of infiltration, interception and surface storage. All infiltrated water is stored in M_R until the entire 25 mm capacity is filled, at which point any additional infiltrated water is transferred directly to M_L (Haan, 1972). When both storage are filled to their capacity, all precipitation is assumed to be runoff.

The surface runoff volume, (Q_s) was determined from

$$Q_s = (P - f)\,t \quad \text{for } P > f \tag{9}$$

$$Q_s = 0 \quad \text{for } P \leq f \tag{10}$$

where t is the time increment involved (hr).

Deep seepage (SP) or water that does not appear as streamflow within the watershed was determined from :

$$SP = S\,(M_L / C) \tag{11}$$

where S is the maximum possible seepage rate (mm per day).

A certain amount of return flow (Q_R) is allowed within the watershed and was calculated from :

$$Q_R = F \times SP \tag{12}$$

where F is a constant defining the fraction of seepage that becomes runoff.

The total runoff (Q_T) is then equal to the sum of the surface runoff and the return flow. It written as :

$$Q_T = Q_S + Q_R \tag{13}$$

Parameter Optimization

This model contains four parameters that must be estimated. The four parameters are:

f_{max} = maximum infiltration rate (mm/hr),
S = maximum rate of seepage from the soil water zone (mm/day),
C = maximum soil water storage capacity (mm),
F = fraction of seepage from the soil water zone that becomes streamflow.

The objective function used to optimize the runoff volume parameters was to minimize the quantity of Z

Where :

$$Z = \sum_{1}^{NMO} (ROOBS\ (I) - ROHAT\ (I))^2 \qquad (14)$$

NMO = Number of months of record used in the optimization
ROHAT (I) = predicted runoff for the I^{th} month
ROOBS (I) = observed runoff for the I^{th} month

The procedure used for optimization is a simple univariate technique. The program requires initial estimates for the parameters and the increment size to be used in changing the value of each of the parameters. The process starts by calculating the value of the objective function at the initial parameter estimates. Next, the value of f_{max} is changed by one increment, while the other parameter values are remaining constant. The objective function is recomputed, and if after the first step the objective function does not improve, a step in the opposite direction is tried.

This procedure is repeated for the values of S, C, and F being varied one at a time. After all four variables have been operated on, the entire process can be repeated for as many iterations as desired. This procedure is continued until the change in the objective function is less than 0.001 * NMO/ 2 (Haan, 1972).

RESULTS AND DISCUSSION

General Description of the Watershed

This study was conducted in Pogung-Code and Pulo-Opak sub watersheds. (Figure 2 and 3). These two sub watersheds were selected on the basis of available data, which were important in attaining the objectives of the study. The physical characteristics of the watershed are summarized in Table 1.

Table 1. Characteristics of the sub watersheds being studied

NO	CHARACTERISTICS	POGUNG-CODE	PULO-OPAK
1	Drainage density (km/km^2)	1.69	1.68
2	Main stream length (km)	25.14	21.61
3	Forest (ha)	384.41	59.12
4	Shrub (ha)	78.90	518.81
5	Plantation (ha)	208.09	170.87
6	Resettlement (ha)	479.54	825.39
7	Grass land (ha)	5.63	243.42
8	Irrigated land (ha)	1,370.41	1,781.81
9	Dry land (ha)	274.79	1,256.95
	Total area (ha)	2,801.77	4,856.37

Source : Interpretation of Land Use and Landsat Image of Yogyakarta (2000)

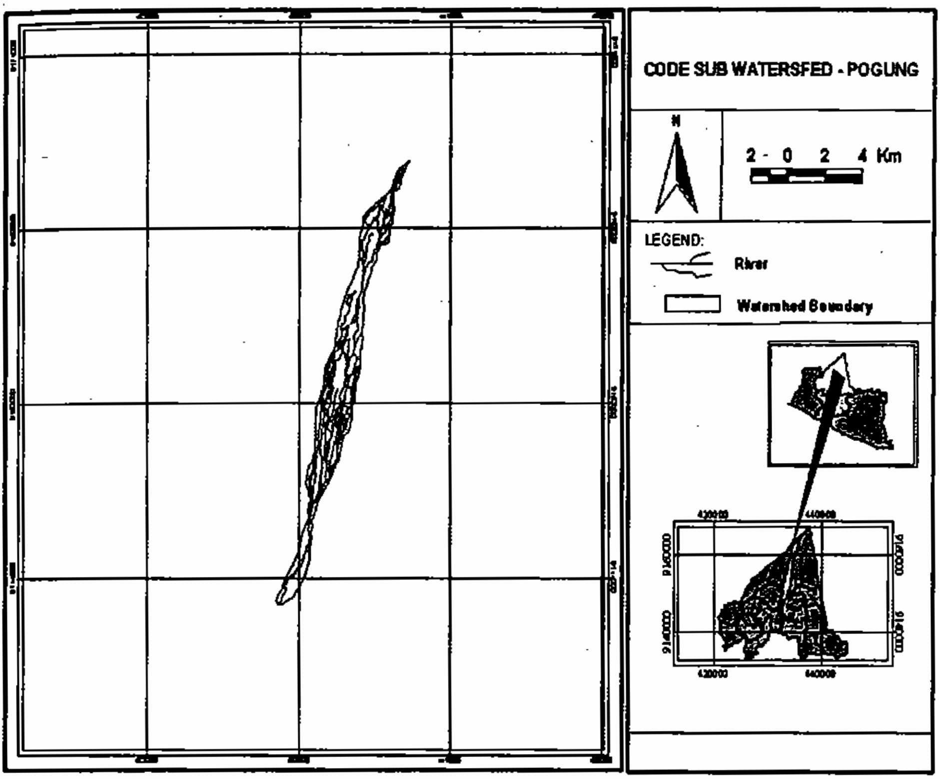

Figure 2. Code sub watershed - Pogung

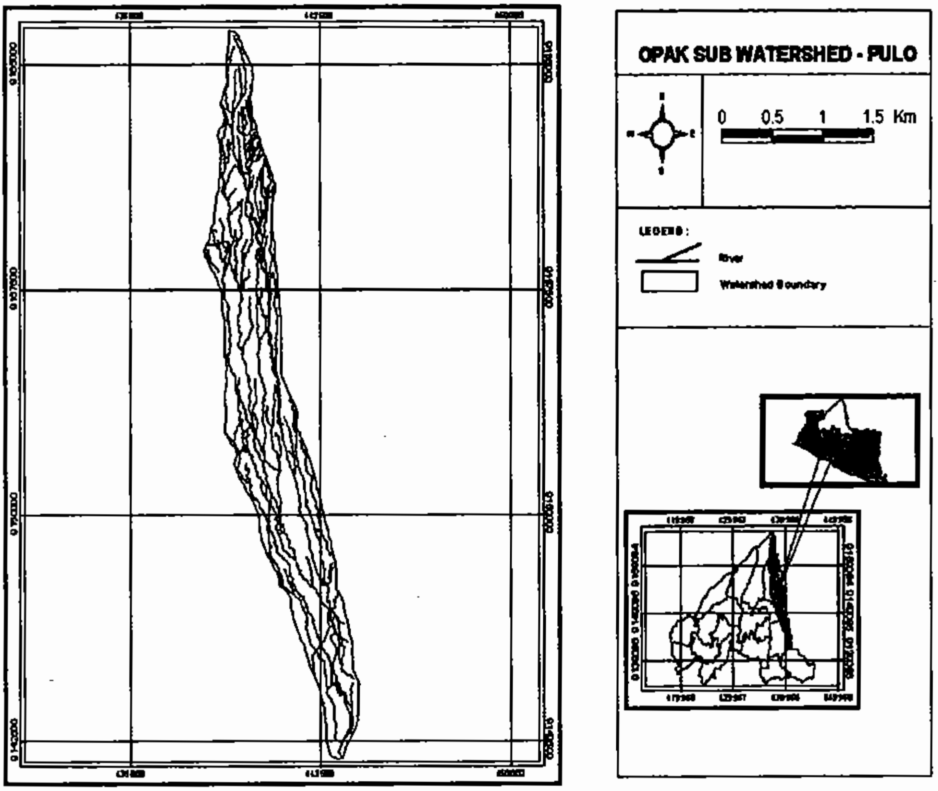

Figure 3. Opak sub watershed - Pulo

Watershed Parameters

The model parameters that have to be estimated are (i) maximum rate of seepage from the soil water zone in mm per day (S), (ii) maximum soil moisture less readily available for evapotranspiration in mm (C), (iii) fraction of seepage from the soil water zone that becomes runoff (F), and (iv) maximum infiltration rate in mm per hour (f_{max}).

To minimize bias as possible in the parameter estimation process, the following procedures were used to obtain the optimal values of the four model parameters :

1) the model parameters were estimated using the subroutine optimum.
2) the entire historical streamflow record was simulated using the parameters obtained in step 1;
3) the two contiguous years that produced the poorest results from simulation in the step 2 were selected and optimal parameter values for these two years were determined using model optimization routine;
4) an average set of parameters was calculated based on the optimal parameters obtained from step 1 and 3;
5) the parameters from step 4 were then taken as the best parameter estimates and were used in the final model evaluation.

The first two years of historical streamflow record (1993 – 1994) were used to determine the model parameters for Pogung-Code sub watershed.

A larger step size was used initially to obtain the preliminary estimates of the four-parameters. Furthermore, the smaller step sizes were used with less number of iterations until the optimal values of the parameters were obtained.

Following step 1, it is found that the model parameters for Pogung-Code sub watershed obtained were S = 0.74 mm per day, C = 115.41 mm, F = 0.61, and f_{max} = 5.22mm per hour. These parameters were used to simulate the entire streamflow records of Pogung-Code sub watershed (1993 - 2002). The two contiguous years that produced the poorest results based on these parameters were 1998 and 1999.

Another set of the optimal parameters were re-determined using the record of this period. The optimal parameters obtained using these data were as follows : S = 0.66 mm per day, C= 109.23 mm, F=0.65, and f_{max}= 5.54 mm per hour.

The final set of parameters considered as the set of parameters was obtained by taking the average values of the results in step 1 and step 3. Those were S=0.70 mm per day, C= 112.32 mm, F=0.63, and f_{max}= 5.38 mm per hour .

Following the procedure described above, the optimal set of parameters for Pulo-Opak sub watershed were S= 2.33 mm per day, C=86.72 mm, F=0.26, and f_{max}= 5.68mm per hour (Table 2). These parameters were used to simulate the entire streamflow record for both sub watersheds.

Table 2. Values of the model parameters of the watershed being studied

SUB WATERSHEDS	OPTIMAL PARAMETERS			
	S (mm/day)	C (mm)	F	f_{max} (mm/hr)
Pogung-Code	0.70	112.32	0.63	5.38
Pulo-Opak	2.33	86.72	0.26	5.68

Observed and Simulated Runoff

Runoff values were computed on a daily basis, however, monthly runoff values were used to statistically test the results. As it was discussed earlier, four years of streamflow record were used to estimate the parameters of each watershed, and these were used to simulate 10 years of streamflow record for Pogung-Code and Pulo-Opak sub watersheds, respectively.

The results of the simulated runoff for both watersheds are summarized in Table 3. The average yearly simulated runoff for Pogung-Code sub watershed was 0.91 m^3 per second or 4.6 percent greater than the observed runoff. On the other hand, the annual simulated runoff value for Pulo-Opak sub watershed was 1.12 m^3 per second (5.7 percent greater than the observed runoff).

Table 3. Summary of results of the monthly mean values of observed and simulated runoff (1993 – 2002)

MONTH	POGUNG-CODE		PULO-OPAK	
	Obs (m^3/sec)	Sim (m^3/sec)	Obs (m^3/sec)	Sim (m^3/sec)
Jan	1.08	1.23	1.77	1.68
Feb	1.25	1.36	1.82	1.79
Mar	1.72	1.61	1.81	1.94
Apr	1.48	1.54	0.97	1.20
May	0.88	1.05	0.68	0.81
Jun	0.63	0.56	0.63	0.62
Jul	0.36	0.30	0.58	0.69
Aug	0.29	0.26	0.59	0.72
Sep	0.48	0.54	0.60	0.56
Oct	0.51	0.65	0.94	0.86
Nov	0.68	0.51	1.13	1.26
Dec	1.02	1.32	1.24	1.32
TOT	10.38	10.93	12.76	13.45
AVE	0.87	0.91	1.06	1.12

Obs : Observed runoff

Sim : Simulated runoff

Statistical Analysis

The evaluation of a model generally involves comparing the model output with observed data. Two statistical parameters were calculated in order to summarize the comparison between observed and simulated monthly runoff. These statistical parameters are : (1) the correlation coefficient (**r**) between observed and simulated monthly flow, and (2) test of significance between the means of observed and simulated monthly flow (T).

The ability of the model to simulate streamflow must be judged by simultaneous comparison of **r**, and T as well as a visual interpretation that relates observed and simulated runoff values, as presented in Figure 4.

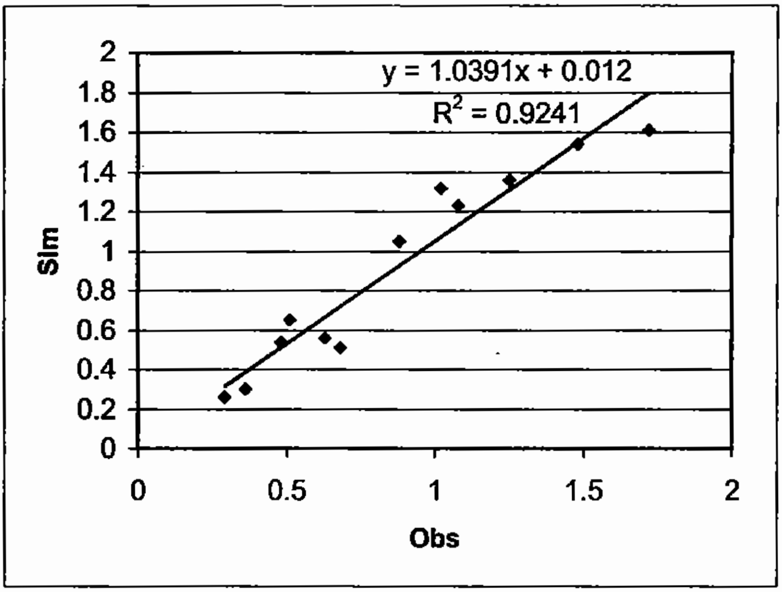

Figure 4. Comparison of observed and simulated monthly runoff for Pogung sub watershed (1993 – 2002)

The statistical analysis shows that there was a high correlation between the observed and simulated runoff for both watersheds. The values of the computed **r** for the entire streamflow record were greater than the tabular **r** with (n-2) degrees of freedom at 5 percent level of significance. This indicates that the observed and simulated runoff were associated with one another. In a linear way, it can be said that high observed values of runoff were associated with high simulated runoff and vice versa. Comparison of the mean values of observed and simulated runoff showed that the computed T values were located within the region of tabular T, with (n-1) degrees of freedom at 95% level of confidence interval. This means that there was no significant difference between the mean values of observed and simulated runoff for both watersheds (Table 4).

Table 4. Test of the validity of the model based on the mean values of observed and simulated runoff

SUB WATERSHEDS	MEAN RUNOFF Obs (m^3/sec)	Sim (m^3/sec)	CORR. COFF. (R)	T
Pogung-Code	0.87	0.91	0.96	-1.021
Pulo-Opak	1.06	1.12	0.93	-0.548

r: r_{tab} *with df (10, 0.05) = 0.576*
T: T_{tab} *with df (11, 0.025) = -2.593 <T<2.593*

CONCLUSION

The model parameters obtained for Pogung-Code sub watershed were S = 0.70 mm per day, C = 112.32 mm, F = 0.63, and f_{max} = 5.38 mm per hour. For Pulo-Opak sub watershed these were S = 2.33 mm per day, C = 86.72 mm, F = 0.26, and f_{max} = 5.68 mm per hour.

The simulation results indicate that there is no significant difference between the means of observed and simulated runoff based on the two-tailed test of significance with 95 percent level of confidence interval. The correlation coefficient for both watersheds were quite high. This implies that the observed and simulated runoff were highly associated with one another.

Generally speaking, the model was able to successfully duplicate the behaviors of observed runoff through a parameter optimization process. However the smaller watershed's (Pogung-Code) model gave a good result compared to the bigger one's (Pulo-Opak).

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