AJCHE 2011, Vol. 11, No. 1, 16 – 21

Two-degree-of-freedom Controller Design for Uncertain Processes Using Input/output Linearization Control Technique

Pisit Sukkarnkha*
Chanin Panjapornpon
[1] Center for Petroleum, Petrochemicals, and Advanced Materials, Department of Chemical Engineering, *Faculty of Engineering, Kasetsart University, Bangkok, 10900, Thailand*
**e-mail:* fengcnp@ku.ac.th

In this work, a new control method for uncertain processes is developed based on two-degree-of-freedom control structure. The setpoint tracking controller designed by input/output linearization technique is used to regulate the disturbance-free output and the disturbance rejection controller designed is designed by high-gain technique. The advantage of two-degree-of-freedom control structure is that setpoint tracking and load disturbance rejection controllers can be designed separately. Open-loop observer is applied to provide disturbance-free response for setpoint tracking controller. The process/disturbance-free model mismatches are fed to the disturbance rejection controller for reducing effect of disturbance. To evaluate the control performance, the proposed control method is applied through the example of a continuous stirred tank reactor with unmeasured input disturbances and random noise kinetic parametric uncertainties. The simulation results show that both types of disturbances can be effectively compensated by the proposed control method.

Keyword: Two-degree-of-freedom control, Input/output linearization, High-gain compensator, Control of uncertain process, Input uncertainty, Parameter uncertainty

INTRODUCTION

In many practical processes, uncertainties frequently appear due to fluctuations in the process streams, unwell-mixed condition, and sensitivity of measuring instruments. An existence of uncertainty in a process may result in deterioration of robustness and control performance.

Two-degree-of-freedom (2DOF) control is an effective method for handling uncertainties problems especially for the input disturbance. The 2DOF control structure includes two controllers that one is used for setpoint tracking and the other is used for disturbance rejection. The main advantage of 2DOF scheme is that both controllers can be designed independently [3]. The disturbance-free process model is implemented in the control system to calculate disturbance-free states for the setpoint tracking controller and also estimate disturbance-free outputs for comparing with actual output. In past decade, 2DOF control system for uncertain processes have received considerable attention in many research works [1]-[6], [11]. To design the disturbance rejection controller, can be achieved by many techniques, such as proportional-integral-derivative (PID) control [2], artificial neural network (ANN) [1], Internal model control (IMC) approach [3], and H_2 optimal synthesis [4]-[6]. However, the mentioned works developed 2DOF control method in Laplace transform domain that is unsuitable for applying to highly nonlinear system such chemical processes. There are only a few research works developed in nonlinear system, such as the model algorithm control (MAC) based on 2DOF control algorithm [11].

Motivated by the previous works, this work proposes a new control method with 2DOF control structure in nonlinear system. The input/output (I/O) linearization control and high-gain technique are developed to handle the processes with unmeasured input disturbances and parametric uncertainties. The I/O linearizing controller with the disturbance-free model provides the setpoint tracking ability while the high-gain controller compensates the offset caused by the mismatch between the actual outputs and the model. The open-loop observer is applied for estimating the disturbance-free process response.

This paper is organized as follows. The mathematical preliminaries using in the control system design are presented in the first section. Then, the formulation of linearizing feedback controller and the high-gain compensator under the 2DOF structure are discussed. Finally, the applications and performances of the proposed control scheme are illustrated by simulation of continuous stirred tank reactor in the presence of unmeasured input disturbances and parametric uncertainties.

MATHEMATICAL PRELIMINARIES

Consider a process modeled as nonlinear system of the form:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \qquad (1)$$

Where *x* denotes the vector of *n* state variables, *u* denotes the vector of *m* manipulated inputs, and *y* denotes the vector of *m* outputs. The relative order of the output y_i, is denoted by r_i, where r_i is the smallest integer for which $\partial[d^{r_i} y_i / dt^{r_i}]/\partial u \neq 0$.

The following assumptions are made:

- The relative orders, $r_1,\ldots,r_m$, are definite.
- The characteristic matrix of the process is non-singular on *X* × *U*, which is $\frac{\partial}{\partial u} h^r(x,u) \neq 0$.
- The process is locally controllable and observable on $X \times U$.

Let $y_i = h_i(x)$ and define the following notation:

$$\begin{aligned} y_i &= h_i(x) \\ \frac{dy_i}{dt} &= h_i^1(x) \\ &\vdots \\ \frac{d^{r_i-1} y_i}{dt^{r_i-1}} &= h_i^{r_i-1}(x) \\ \frac{d^{r_i} y_i}{dt^{r_i}} &= h_i^{r_i}(x,u) \end{aligned} \qquad (2)$$

Control System Design

The schematic diagram of the proposed control system is shown in Fig. 1, which consists of the setpoint tracking controller, the disturbance rejection controller, and the open-loop observer. The 2DOF control scheme provides the efficient disturbance rejection. The observer is used to estimate the disturbance-free states for the setpoint tracking controller. More details of the proposed control system are given in following subsections.

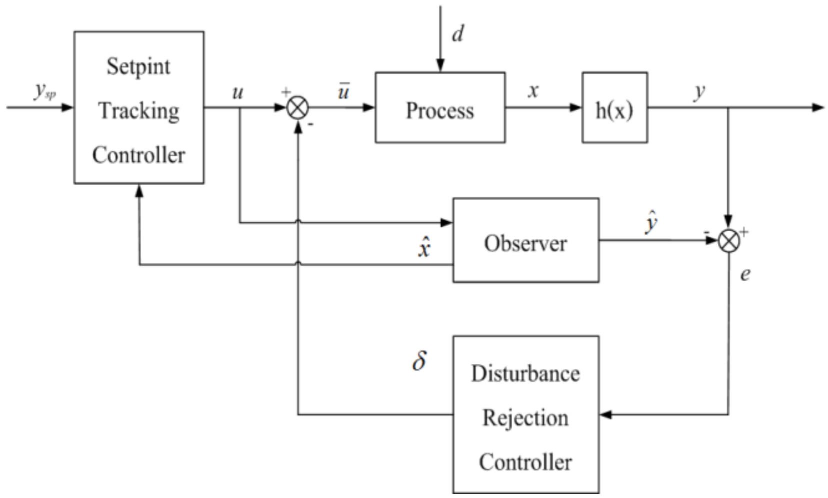

Figure 1. Proposed Control Structure

Setpoint Tracking Controller

Let us request a linear response of the following form for each output

$$\begin{aligned}(\varepsilon_1 \mathrm{D}+1)^{r_1}\, y_1 &= y_{sp,1}\\ &\vdots\\ (\varepsilon_m \mathrm{D}+1)^{r_m}\, y_m &= y_{sp,m}\end{aligned} \qquad (3)$$

where D is the differential operator, $y_{sp,1},\dots,y_{sp,m}$ are the desired setpoints, $\varepsilon_1,\dots,\varepsilon_m$ are tuning parameters that adjust the speed of the responses of the outputs, $y_1,\dots,y_m$, respectively. By substituting the time derivatives of the outputs defined in (2), one obtains

$$\begin{aligned}h_1(x)+\binom{r_1}{1}\varepsilon_1 h_1^1(x)+\cdots+\binom{r_1}{r_1}\varepsilon_1^{r_1} h_1^{r_1}(x,u) &= y_{sp,1}\\ &\vdots\\ h_m(x)+\binom{r_m}{1}\varepsilon_m h_m^1(x)+\cdots+\binom{r_m}{r_m}\varepsilon_m^{r_m} h_m^{r_m}(x,u) &= y_{sp,m}\end{aligned} \qquad (4)$$

where $\begin{pmatrix} a \\ b \end{pmatrix} = \dfrac{a!}{(b-a)!}$

The closed-loop responses of the outputs in (4) can present in the compact form:

$$\begin{aligned}\Phi_1(x,u) &= y_{sp,1}\\ &\vdots\\ \Phi_m(x,u) &= y_{sp,m}\end{aligned} \qquad (5)$$

By solving the equation (5) for *u*, the static feedback controller can be obtained in following form:

$$u = \Psi(x, y_{sp}) \qquad (6)$$

Disturbance Rejection Controller

The disturbance rejection controller is constructed as the following

$$\begin{aligned}\delta_1 &= K_1\left(y_1-\hat{y}_1\right)\\ &\vdots\\ \delta_m &= K_m\left(y_m-\hat{y}_m\right)\end{aligned} \qquad (7)$$

where $\delta_1,\dots,\delta_m$ are the estimated disturbances of the outputs $y_1,\dots,y_m$, $\hat{y}_1,\dots,\hat{y}_m$ are the estimates of disturbance-free process outputs obtained from the observer, and $K_1,\dots,K_m$ is the tuning parameters that should be selected for stabilizing the uncertain processes. The disturbance rejection controller performs to force the process outputs to approach the estimated disturbance-free outputs.

Open-loop Observer

The setpoint tracking controller requires information of the disturbance-free states. To estimate that information, the open-loop observer is applied. The dynamics of open-loop state observer is described by following equation

$$\dot{\hat{x}} = f(\hat{x},u)$$
$$\hat{y} = h(\hat{x}) \quad (8)$$

where $\hat{x}$ is the vectors of estimated states and $\hat{y}$ is the vectors of estimated outputs.

RESULTS AND DISCUSSION

To illustrate the application of the proposed control strategy, a simulation study will be used. Consider a continuous stirred tank reactor (CSTR) with heat removal via cooling jacket shown in Fig. 2. The reaction is exothermic, irreversible, first-order reaction of $A \rightarrow B$. The reactor volume and physical parameters are assumed to be constant and the reactor is operated in perfect mixing condition.

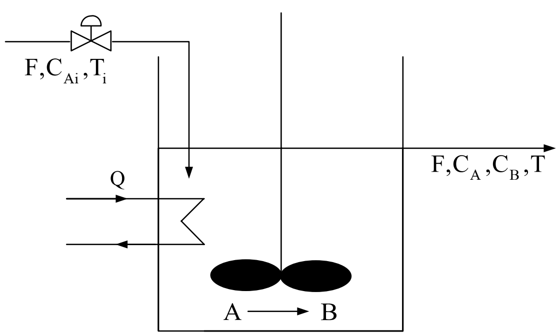

Figure 2. Continuous Stirred Tank Reactor (CSTR)

The mathematical model of the reactor can be expressed by

$$\frac{dC_A}{dt} = -k_0 \exp\left(-\frac{E_a}{RT}\right)C_A + (C_{Ai} - C_A)\frac{F}{V}$$
$$\frac{dT}{dt} = \gamma k_0 \exp\left(-\frac{E_a}{RT}\right)C_A + (T_i - T)\frac{F}{V} + Q \quad (9)$$
$$y = T$$

where C_A is the concentration of A, T is the reactor temperature and F is the inlet volumetric flow rate. The values of the process parameters and nominal operating conditions are given in Table 1. Let $x = [C_A\ T]^T$, $y = T$ and $u = F$. In this example, the reactor temperature is only measurable states.

Table 1. The Value of Parameters

Symbol	Quantity	Value
$C_{A,i}$	Initial concentration of A	12 kmol/m^3
T_i	Initial temperature	300 K
k_0	Arrhenius factor	1.8×10^{-12} hr^{-1}
E_a/R	Activation energy/gas constant	8,100 K
Q	Specific rate of cooling	-90.7 K/hr
V	Volume of reactor	0.1 m^3
γ	Specific heat of reaction	3.9 K.m^3/kmol

For simulation, the proposed control system has been applied to the process. Both controllers are shown in following equations:

Setpoint tracking controller

$$u = \left(\left(\frac{V - \hat{x}_2}{\varepsilon}\right) - \frac{\gamma k_0}{\exp\left(\frac{E_a}{R\hat{x}_2}\right)}\hat{x}_1 - Q \right)\left(\frac{V}{T_i - \hat{x}_2}\right) \quad (10)$$

Disturbance rejection controller

$$\delta = K(y - \hat{x}_2) \quad (11)$$

The controller parameter values are chosen ε = 0.7 and K = 0.6. At first, the desired setpoint of the process is $y_{sp,1}$ = 293.9 K (F_{ss} = 0.45 m^3/hr, $C_{A,ss}$ = 8.4 kmol/m^3, and T_{ss} = 293.9 K), then change to $y_{sp,2}$ = 283.3 K (F_{ss} = 0.38 m^3/hr, $C_{A,ss}$ = 10.2 kmol/m^3, and T_{ss} = 283.3 K).

The performance and the robustness are

tested by two different types of uncertain process, +0.02 m^3/hr load disturbance in inlet flow rate and ± 30% random noise parametric uncertainty in Arrhenius factor (k_0).

Fig. 3 and 4 show the closed-loop responses of the process with constant disturbance at input and random parametric uncertainty, respectively. The results are indicated, both of the setpoint tracking controller and the disturbance rejection controller successfully operate the reactor at the desired setpoints without overshoot. In Fig. 4, the behaviors of F continuously oscillate to maintain T at desired setpoint all the time.

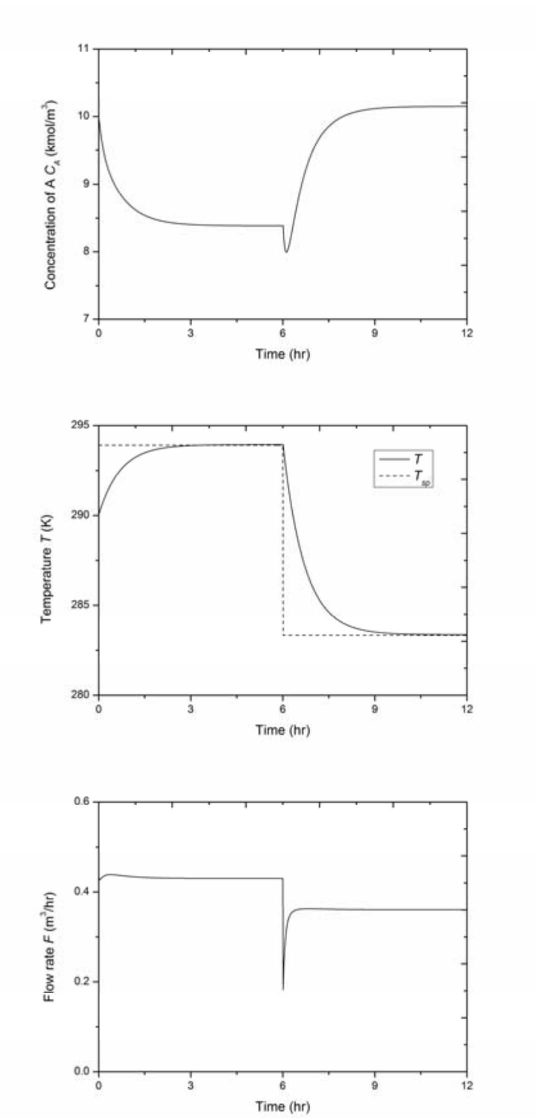

Figure 3. Closed-loop response of unmeasured state variable, controlled output, and manipulated variable for the case of +0.02 m^3/hr load disturbance in inlet flow rate

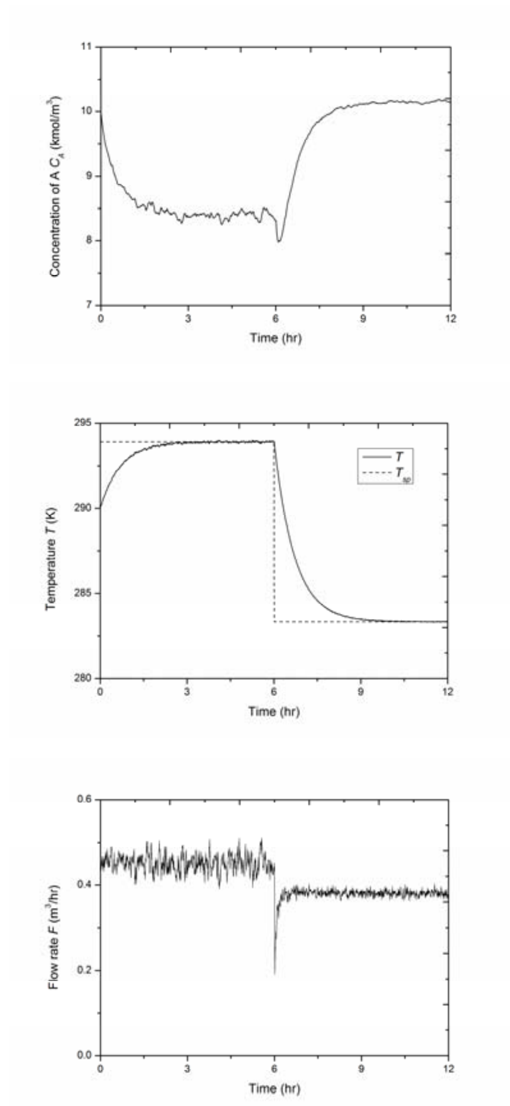

Figure 4. Closed-loop response of unmeasured state variable, controlled output, and manipulated variable for the case of ± 30% random noise parametric uncertainty in Arrhenius factor (k_0).

CONCLUSION

In this work, input-output linearization controller with 2DOF control structure was proposed to handle the nonlinear process with input and parameter uncertainties. With the 2DOF structure, the setpoint tracking and disturbance rejection responses can be independently designed. The I/O linearizing controller provides the tracking ability with the use of a few tuning parameters. The robustness of disturbance rejection is achieved by adjusting the gain of the disturbance rejection controller. The performance of proposed control method is illustrated via its application to a CSTR with constant disturbance in feed flow rate and random noise in the Arrhenius factor. From the results of simulation, it showed that the proposed method is successfully stabilized and maintained the uncertain process at desired condition.

ACKNOWLEDGMENT

This work was financially supported by Kasetsart University Research and Development Institute (KURDI), the Thailand Research Fund (TRF) with the grant number MRG5180221, and National Center of Excellence for Petroleum, Petrochemicals and Advanced Materials. These supports are gratefully acknowledged.

REFERENCES

1) M.H. Tsai and P.C. Tung: A disturbance reduction scheme for linear small delay systems with modeling uncertainties, Journal of Process Control (2010).
2) K.L. Wu, C.C. Yu, and Y.C. Cheng: A two degree of freedom level control, Journal of Process Control, **311–319** (2001) 11.
3) W. Tan: Analysis and design of a double two-degree-of-freedom control scheme, ISA transactions **311-317** (2010) 49.
4) T. Liu, W. Zhang, and D. Gu: Analytical design of two-degree-of-freedom control scheme for open-loop unstable processes with time delay, Journal of Process Control, **559–572** (2005) 15.
5) T. Liu, W. Zhang, and F. Gao: Analytical Two-Degrees-of-Freedom (2-DOF) Decoupling Control Scheme for Multiple-Input-Multiple-Output (MIMO) Processes with Time Delays, Ind. Eng. Chem. Res, **6546–6557** (2007) 46.
6) T. Liu, D. Gu, and W. Zhang: Decoupling two-degree-of-freedom control strategy for cascade control systems, Journal of Process Control, **159–167** (2005) 15.
7) J.M. Kanter, M. Soroush, and W.D. Seider: Nonlinear feedback control of multivariable non-minimum-phase processes, Journal of Process Control, **667–686** (2002) 12.
8) M. Soroush: Nonlinear state-observer design with application to reactors, Chemical Engineering Science, **387–404** (1997) 52.
9) C. Panjapornpon and M. Soroush: Shortest-prediction-horizon non-linear model-predictive control with guaranteed asymptotic stability, International Journal of Control, **1533–1543** (2007) 80.
10) C. Kravaris and M. Soroush: Synthesis of multivariable nonlinear controllers by input/output linearization, AIChE Journal, **249–264** (1990) 36.
11) R.A. Wright and C. Kravaris: Two-degree-of-freedom output feedback controllers for discrete-time nonlinear systems, Chemical Engineering Science, **4676–4688** (2006) 61.