THE ANALITICAL SOLUTIONS OF THE LINEAR TWO CHANNELS DISSIPATION MODEL

Sumardi, Soeparna Darmawijaya, Lina Aryati

Department of Mathematics, Gadjah Mada University, Yogyakarta 55281
E-mail:mas_mardi@yahoo.com

ABSTRACT

The analytical solutions of the linear two channels dissipation model are presented in various forms. First we analyze the particular solutions of this model. Then the model is transformed into the Telegrapher Equation and further into the Klein Gordon Equation for which various families of solutions are known. The Fourier Transform is applied on the Telegrapher Equation, yielding solutions in Fourier representation. Finally we apply the method of characteristics to find solutions of the initial value problem.

Keywords : hyperbolic system, partial differential equation, exact solution

1. INTRODUCTION

Many processes occurring in natural and physical sciences are studied from conservation laws. Conservation laws are just balance laws, equations expressing the fact that in any volume for any quantity the generation, loss, inflow and outflow must have net sum zero. Mathematically, conservation laws usually translate into differential equations. In this paper let us consider two quantities *u(x,t)* and *v(x,t)* that depend on a single spatial variable *x* and on time $t > 0$. Let us think of *u* and *v* as densities or concentrations measured in amount per unit volume that flow at constant or piecewise constant speeds $c_1(x)$ and $c_2(x)$ respectively, and that have interaction with each other through a linear relaxation term. Or we may think of *u* and *v* as being the temperature of two fluids that flow on either side of a membrane through which heat is being exchanged. This type of situations leads to the following set of equations:

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} + c_1(x)\frac{\partial u(x,t)}{\partial x} &= \alpha\, v(x,t) - \alpha\, u(x,t) \\ \frac{\partial v(x,t)}{\partial t} + c_2(x)\frac{\partial v(x,t)}{\partial x} &= \alpha\, u(x,t) - \alpha\, v(x,t) \end{aligned} \qquad (1)$$

The constant α is the exchange coefficient.

The model can be found e.g. in Beckum (2003) and the application of the model on heat and mass transfer phenomena in a medium can be found e.g. in Gupalo(1995). A numerical approach (finite difference method) for the above equations under boundary and initial conditions in the case $c_1(x) = -c_2(x) = 1$ can be found e.g. in Sumardi (2005), including a rigorous proof of convergence. Sumardi (2007) also did numerical calculations by the Immersed Interface Method in that $c_1(x)$ and $c_2(x)$ are piecewise constants. Mascia (1996) obtained uniform estimate on the derivatives of solutions. However, it will be interesting if we could find its analytic solution.

In the present paper, we present the analytical solution of the model in various forms in the case that the velocities may be of the same sign (co-current flows) or of different sign (counter-current flows). One of them may even be zero. The model can as well be transformed into a second order partial differential equation then to the Telegrapher Equation, and further into the Klein Gordon Equation. Discussion of the methods for this type of PDEs are available too. See e.g. Polyanin (2002), Evans (1998), and Pinsky (1998).

2. PARTICULAR SOLUTIONS

Consider the linear two channels dissipation model equation (1). Without loss of generality we assume from now on that c_1 is positive and that $|c_2| \le c_1$

$$\begin{aligned}\frac{\partial u(x,t)}{\partial t} + c_1 \frac{\partial u(x,t)}{\partial x} &= \alpha\, v(x,t) - \alpha\, u(x,t) \\ \frac{\partial v(x,t)}{\partial t} + c_2 \frac{\partial v(x,t)}{\partial x} &= \alpha\, u(x,t) - \alpha\, v(x,t)\end{aligned} \qquad (2)$$

2.1 Anzatz Method

First we find the particular solutions of the linear two channels dissipation model by Anzatz method. Suppose the particular solution is a pair of the exponential functions of the form

$$\begin{aligned} u(x,t) &= Ae^{kx+mt} \\ v(x,t) &= Be^{kx+mt} \end{aligned} \qquad (3)$$

Then we find *A*, *B* and *m* in order to satisfy Equation (2), we obtain system equation

$$\begin{aligned}(m + kc_1 + \alpha)A - \alpha B &= 0 \\ -\alpha A + (m + kc_2 + \alpha)B &= 0\end{aligned} \qquad (4)$$

To get a non trivial solution, we set

$$\begin{vmatrix} m + kc_1 + \alpha & -\alpha \\ -\alpha & m + kc_2 + \alpha \end{vmatrix} = 0 \qquad (5)$$

so we obtain the quadratic equation

$$m^2 + (kc_1 + kc_2 + 2\alpha)m + c_1c_2 + \alpha k(c_1 + c_2) = 0 \qquad (6)$$

that has two distinct real square roots. The discriminant of the quadratic equation is $D = k^2(c_1 - c_2)^2 + 4\alpha^2$. Hence we obtain

$$m_{1,2} = \frac{-kc_1 - kc_2 - 2\alpha \pm \sqrt{D}}{2}. \qquad (7)$$

Substitute (7) into (4), we obtain the relationship between the constants *A* and *B*:

$$B = \frac{k(c_1 - c_2) \pm \sqrt{D}}{2\alpha} A, \qquad (8)$$

so we obtain particular solution:

$$\begin{aligned} u(x,t) &= Ae^{kx + \frac{-c_1 - c_2 - 2\alpha \pm \sqrt{D}}{2}t} \\ v(x,t) &= \frac{k(c_1 - c_2) \pm \sqrt{D}}{2\alpha} Ae^{kx + \frac{-c_1 - c_2 - 2\alpha \pm \sqrt{D}}{2}t} \end{aligned} \qquad (9)$$

2.2 Transform to second order equation

We can also obtain the particular solutions of the two channels dissipation model by elimination method, so we get a second order hyperbolic partial differential equation.

Let equations (2) be written as follow:

$$\begin{aligned} \left(\frac{\partial}{\partial t} + c_1 \frac{\partial}{\partial x} + \alpha\right)u(x,t) - \alpha\, v(x,t) &= 0 \\ -\alpha\, u(x,t) + \left(\frac{\partial}{\partial t} + c_2 \frac{\partial}{\partial x} + \alpha\right)v(x,t) &= 0 \end{aligned} \qquad (10)$$

If the first equation multiplies by $\frac{\partial}{\partial t} + c_2 \frac{\partial}{\partial x} + \alpha$ and the second equation multiplies by α then we can eliminate *v(x,t)*, so we obtain:

$$\frac{\partial^2 u}{\partial t^2} + (c_1 + c_2)\frac{\partial^2 u}{\partial t \partial x} + c_1c_2 \frac{\partial^2 u}{\partial x^2} + 2\alpha \frac{\partial u}{\partial t} + \alpha(c_1 + c_2)\frac{\partial u}{\partial x} = 0 \qquad (11)$$

In order to solve equation (11) consider the changes variables in the case that $c_1 \ne c_2$

$$\begin{aligned} \eta &= x - \frac{1}{2}(c_1 + c_2)t \\ \tau &= \frac{1}{2}(c_1 - c_2)t \end{aligned} \qquad (12)$$

Using the Chain rule for transforming the partial derivatives of the two variable functions, we have the Telegrapher equation:

$$\frac{\partial^2 u}{\partial \tau^2} + \frac{4\alpha}{c_1 - c_2}\frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial \eta^2} = 0. \qquad (13)$$

To put this in more convenient form, let

$$\beta = \frac{2\alpha}{c_1 - c_2}, \qquad (14)$$

resulting in the equation

$$\frac{\partial^2 u}{\partial \tau^2} + 2\beta \frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial \eta^2} = 0. \qquad (15)$$

The Telegrapher equation is transformed in the Klein Gordon equation by

$$u(\eta,\tau) = e^{-\beta\tau} w(\eta,\tau) \qquad (16)$$

so we obtain

$$\frac{\partial^2 w}{\partial \tau^2} - \frac{\partial^2 w}{\partial \eta^2} - \beta^2 w = 0. \qquad (17)$$

Polyanin (2002) wrote many particular solutions of the Klein Gordon equation (17):

$$w(\eta,\tau) = e^{\pm\beta\tau}(A\eta + B) \qquad (18)$$

$$w(\eta,\tau) = \cos(\eta\sqrt{\beta^2 + \mu^2}) \times (A\cos(\mu\tau) + B\sin(\mu\tau)) \qquad (19)$$

$$w(\eta,\tau)=\sin(\eta\sqrt{\beta^2+\mu^2})\times (A\cos(\mu\tau)+B\sin(\mu\tau)) \quad (20)$$

$$w(\eta,\tau)=e^{\pm\tau\sqrt{\beta^2-\mu^2}}\times (A\cos(\mu\eta)+B\sin(\mu\eta)) \quad (21)$$

$$w(\eta,\tau)=e^{\pm\tau\sqrt{\beta^2-\mu^2}}(Ae^{\mu\eta}+Be^{-\mu\eta}) \quad (22)$$

$$w(\eta,\tau)=AI_0(\xi)+BK_0(\xi),\quad \xi=\beta\sqrt{(\tau+C_1)^2-(\eta+C_2)^2} \quad (23)$$

where μ, A, B, C_1 and C_2 are arbitrary constants, $I_0(\xi)$ and $K_0(\xi)$ are the modified Bessel functions. Then we work our way backwards with substituting the inverse transformation (10), (12), (14) and (16) we obtain the result below.

From the equation (18), we have

$$u(x,t)=A\left(x-\frac{t}{2}(c_1+c_2)\right)+B$$
$$v(x,t)=A\left(x-\frac{t}{2}(c_1+c_2)+\frac{c_1-c_2}{2\alpha}\right)+B$$

, (24)

$$u(x,t)=e^{-2\alpha t}\left(A(x-\frac{t}{2}(c_1+c_2))+B\right)$$
$$v(x,t)=-e^{-2\alpha t}\times\left(A(x-\frac{t}{2}(c_1+c_2)+\frac{c_1-c_2}{2\alpha})+B\right) \quad (25)$$

From the equation (19), we have the particular solution:

$$u(x,t)=e^{-\alpha t}\cos\left((x-\frac{(c_1+c_2)t}{2})\sqrt{\beta^2+\mu^2}\right)\times\left(\cos(\mu\frac{(c_1-c_2)t}{2})A+\sin(\mu\frac{(c_1-c_2)t}{2})B\right)$$

$$\begin{aligned}v(x,t)=&A\frac{(c_2-c_1)\mu}{2\alpha}e^{-\alpha t}\sin(\mu\frac{(c_1-c_2)t}{2})\times\cos\left((x-\frac{(c_1+c_2)t}{2})\sqrt{\beta^2+\mu^2}\right)\\&+A\left(\frac{c_1+c_2}{2\alpha}\sqrt{\beta^2+\mu^2}-\frac{1}{\alpha}\right)e^{-\alpha t}\cos(\mu\frac{(c_1-c_2)t}{2})\times\sin\left((x-\frac{(c_1+c_2)t}{2})\sqrt{\beta^2+\mu^2}\right)\\&+B\frac{(c_1-c_2)\mu}{2\alpha}e^{-\alpha t}\cos(\mu\frac{(c_1-c_2)t}{2})\times\cos\left((x-\frac{(c_1+c_2)t}{2})\sqrt{\beta^2+\mu^2}\right)\\&+B\left(\frac{c_1+c_2}{2\alpha}\sqrt{\beta^2+\mu^2}-\frac{1}{\alpha}\right)e^{-\alpha t}\times\sin(\mu\frac{(c_1-c_2)t}{2})\sin\left((x-\frac{(c_1+c_2)t}{2})\sqrt{\beta^2+\mu^2}\right)\end{aligned} \quad (26)$$

Similarly we can find the other particular solutions of the linear two channels dissipation model by using the equations (20)– (23).

3. THE INITIAL VALUE PROBLEM

For the section we will investigate the initial value problem of the linear two channels dissipation model. Let

$$\frac{\partial u(x,t)}{\partial t}+c_1\frac{\partial u(x,t)}{\partial x}=\alpha\, v(x,t)-\alpha\, u(x,t)$$
$$\frac{\partial v(x,t)}{\partial t}+c_2\frac{\partial v(x,t)}{\partial x}=\alpha\, u(x,t)-\alpha\, v(x,t) \quad (27)$$

$$u(x,0)=u_0(x),\ v(x,0)=v_0(x)\ x\in R, t\geq 0 \quad (28)$$

3.1 Case $c_1=c_2$

For the special case we take $c_1=c_2=c$, so we have

$$\begin{aligned}\frac{\partial u(x,t)}{\partial t}+c\frac{\partial u(x,t)}{\partial x}&=\alpha v(x,t)-\alpha u(x,t)\\ \frac{\partial v(x,t)}{\partial t}+c\frac{\partial v(x,t)}{\partial x}&=\alpha u(x,t)-\alpha v(x,t)\end{aligned} \quad (29)$$

Adding two equations (29) we have advection equation, which have solution:

$$u(x,t)+v(x,t)=F(x-ct) \quad (30)$$

where F is an arbitrary function. Substitute (30) in the first equation (29) we obtain

$$\begin{aligned}&\frac{\partial u(x,t)}{\partial t}+c\frac{\partial u(x,t)}{\partial x}\\ &=-2\alpha u(x,t)+\alpha F(x-ct)\end{aligned} \quad (31)$$

then solve the equation (31), hence we have general solution of the equation (29):

$$\begin{aligned}u(x,t)&=\frac{1}{2}\left(F(x-ct)+e^{-2\alpha t}G(x-ct)\right)\\ v(x,t)&=\frac{1}{2}\left(F(x-ct)-e^{-2\alpha t}G(x-ct)\right)\end{aligned} \quad (32)$$

Applying the initial condition (28), we have the solution of the initial value problem (27)-(28) in the case that $c_1=c_2=c$:

$$\begin{aligned}u(x,t)&=\frac{1}{2}\left(u_0(x-ct)+v_0(x-ct)\right)\\ &\quad+\frac{1}{2}e^{-2\alpha t}\left(u_0(x-ct)-v_0(x-ct)\right)\\ v(x,t)&=\frac{1}{2}\left(u_0(x-ct)+v_0(x-ct)\right)\\ &\quad-\frac{1}{2}e^{-2\alpha t}\left(u_0(x-ct)-v_0(x-ct)\right)\end{aligned}. \quad (33)$$

From the solution (33) can be interpreted that the solutions $u(x,t)$ and $v(x,t)$ are the average of the initial signal $u_0(x)$ and $v_0(x)$ with dissipative the difference of the initial signal that shifted to the right by the amount ct. For a long time, $u(x,t)$ tends $v(x,t)$.

3.2 Case $c_1 \neq c_2$

In the case that $c_1 \neq c_2$ using elimination method and transformation to variable of the equation (15), we get the new initial value problem in the Telegrapher equation:

$$\frac{\partial^2 u}{\partial \tau^2}+\frac{4\alpha}{c_1-c_2}\frac{\partial u}{\partial \tau}-\frac{\partial^2 u}{\partial \eta^2}=0 \quad (34)$$

$$\begin{aligned}u(\eta,0)\quad&=f(\eta)=u_0(\eta)\\ \left.\frac{\partial u}{\partial \tau}\right|_{\tau=0}&=g(\eta)\\ &=\frac{2}{c_1-c_2}\left(\alpha v_0(\eta)-\alpha u_0(\eta)-c_1u_0'(\eta)\right)\end{aligned} \quad (35)$$

To solve the initial value problem (34)-(35), we look for $u(\eta,\tau)$ in terms of its Fourier Transform

$$u(\eta,\tau)=\int_{-\infty}^{\infty}U(\mu,\tau)e^{i\eta\mu}d\mu \quad (36)$$

$$U(\mu,\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}u(\eta,\tau)e^{-i\eta\mu}d\eta \quad (37)$$

and formally apply the operations implied by (36.a)

$$\begin{aligned}&\frac{\partial^2 u}{\partial \tau^2}+\frac{4\alpha}{c_1-c_2}\frac{\partial u}{\partial \tau}-\frac{\partial^2 u}{\partial \eta^2}\\ &=\int_{-\infty}^{\infty}\left(\frac{\partial^2 U}{\partial \tau^2}+\frac{4\alpha}{c_1-c_2}\frac{\partial U}{\partial \tau}+\mu^2U\right)e^{i\eta\mu}d\mu\end{aligned}.$$

(38)

Therefore we solve the ordinary differential equation

$$\frac{\partial^2 U}{\partial \tau^2}+\frac{4\alpha}{c_1-c_2}\frac{\partial U}{\partial \tau}+\mu^2U=0 \quad (39)$$

with initial conditions

$$\begin{aligned}U(\mu,\tau)&=F(\mu)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(\eta)e^{-i\eta\mu}d\eta\\ \left.\frac{\partial U(\mu,\tau)}{\partial \mu}\right|_{\tau=0}&=G(\mu)=\frac{1}{2\pi}\int_{-\infty}^{\infty}g(\eta)e^{-i\eta\mu}d\eta\end{aligned} \quad (40)$$

We seek a solution (39) of the form

$$U(\mu,\tau)=Be^{\gamma\tau}, \qquad B,\gamma\in C. \quad (41)$$

$$\gamma=-\left(\frac{4\alpha}{c_1-c_2}\right)\pm\sqrt{\left(\frac{4\alpha}{c_1-c_2}\right)^2-\mu^2}.$$

Plugging into (39), we deduce

$\gamma^2+\frac{4\alpha}{c_1-c_2}\gamma+\mu^2=0$, hence

Consequently

$$U(\mu,\tau)=\begin{cases}e^{\left(\frac{4\alpha}{c_1-c_2}\right)\tau}\left(B_1(\mu)e^{\gamma(\mu)\tau}+B_2(\mu)e^{-\gamma(\mu)\tau}\right) & \text{if } |\mu|\le\left|\frac{4\alpha}{c_1-c_2}\right|\\ e^{\left(\frac{4\alpha}{c_1-c_2}\right)\tau}\left(B_1(\mu)e^{i\delta(\mu)\tau}+B_2(\mu)e^{-i\delta(\mu)\tau}\right) & \text{if } |\mu|>\left|\frac{4\alpha}{c_1-c_2}\right|\end{cases}$$

(42)

for $\gamma(\mu)=\sqrt{\left(\frac{2\alpha}{c_1-c_2}\right)^2-\mu^2}$,

$\delta(\mu)=\sqrt{\mu^2-\left(\frac{2\alpha}{c_1-c_2}\right)^2}$ where $B_1(\mu)$ and $B_2(\mu)$ are selected so that

$$F(\mu)=B_1(\mu)+B_2(\mu)$$

and

$$G(\mu)=\begin{cases} B_1(\mu)\left(\gamma(\mu)-\left(\frac{2\alpha}{c_1-c_2}\right)\right)+ \\ B_2(\mu)\left(-\gamma(\mu)-\left(\frac{2\alpha}{c_1-c_2}\right)\right) & \text{if } |\mu|\le\left|\frac{2\alpha}{c_1-c_2}\right| \\ B_1(\mu)\left(i\delta(\mu)-\left(\frac{2\alpha}{c_1-c_2}\right)\right)+ \\ B_2(\mu)\left(-i\delta(\mu)-\left(\frac{2\alpha}{c_1-c_2}\right)\right) & \text{if } |\mu|>\left|\frac{2\alpha}{c_1-c_2}\right| \end{cases}. \tag{43}$$

We thereby obtain the exact solution of the initial value problem of the linear two channels dissipation model in the Fourier representation:

$$u(\eta,\tau)=\int_{|\mu|\le\left|\frac{2\alpha}{c_1-c_2}\right|} e^{\left(\frac{2\alpha}{c_1-c_2}\right)\tau} e^{i\eta\mu}\times \left(B_1(\mu)e^{\gamma(\mu)\tau}+B_2(\mu)e^{-\gamma(\mu)\tau}\right)d\mu + \int_{|\mu|>\left|\frac{2\alpha}{c_1-c_2}\right|} e^{\left(\frac{2\alpha}{c_1-c_2}\right)\tau} e^{i\eta\mu}\times \left(B_1(\mu)e^{i\delta(\mu)\tau}+B_2(\mu)e^{-i\delta(\mu)\tau}\right)d\mu. \tag{44}$$

Then we work our way backwards with substituting the inverse transformation (15) we obtain the result:

$$u(x,t)=\int_{|\mu|\le\left|\frac{2\alpha}{c_1-c_2}\right|} e^{-\alpha t} e^{i(x-\frac{t}{2}(c_1+c_2))\mu}\times \left(B_1(\mu)e^{\frac{t}{2}\sqrt{4\alpha^2-\mu^2(c_1+c_2)^2}}+B_2(\mu)e^{-\frac{t}{2}\sqrt{4\alpha^2-\mu^2(c_1+c_2)^2}}\right)d\mu + \int_{|\mu|>\left|\frac{2\alpha}{c_1-c_2}\right|} e^{\left(\frac{4\alpha}{c_1-c_2}\right)\tau} e^{i\frac{t}{2}\sqrt{\mu^2(c_1+c_2)^2-4\alpha^2}}\times \left(B_1(\mu)e^{i\frac{t}{2}\sqrt{\mu^2(c_1+c_2)^2-4\alpha^2}}+B_2(\mu)e^{-i\delta(\mu)\tau}\right)d\mu \tag{45}$$

and

$$v(x,t)=\frac{1}{\alpha}\left(\frac{\partial u(x,t)}{\partial t}+c_1\frac{\partial u(x,t)}{\partial x}+\alpha u(x,t)\right) . \tag{46}$$

It is very necessary to prove the existence and uniqueness solution of the initial value problem of two channel dissipation model.

3.3 Characteristics Method

As a third method we treat the initial value problem by characteristic method. The fundamental idea with hyperbolic equations is the notion of the characteristics, curves in space time along which these signal are propagated. In the curves along the partial differential equations can be reduced to simple form for example, system of ordinary differential equations. The characteristic are the curves along which information is transmitted in the system.

Let $(x,t)\in\Re^2$ and every $s\in\Re$ that defined by

$$\begin{aligned} w(s) &= u(x+c_1s,t+s) \\ z(s) &= v(x+c_2s,t+s) \end{aligned}. \tag{47}$$

Furthermore we obtain

$$\begin{aligned}\frac{dw(s)}{ds} &= u_t(x+c_1s,t+s)+c_1u_x(x+c_1s,t+s) \\ &= \alpha\big(v(x+c_1s,t+s)-u(x+c_1s,t+s)\big)\end{aligned} \tag{48}$$

$$\begin{aligned}\frac{dz(s)}{ds} &= v_t(x+c_2s,t+s)+c_2v_x(x+c_2s,t+s) \\ &= \alpha\big(u(x+c_2s,t+s)-v(x+c_2s,t+s)\big)\end{aligned} \tag{49}$$

The equation (46) is integrated from $s=-t$ to $s=0$, we obtain

$$\int_{-t}^{0} \frac{dw(s)}{ds} ds = \int_{-t}^{0} \alpha\big(v(x+c_1 s, t+s) - u(x+c_1 s, t+s)\big) ds$$

$$w(0) - w(-s) = \int_{0}^{t} \alpha\big(v(x+c_1(s-t), s) - u(x+c_1(s-t), s)\big) ds \tag{50}$$

and because of

$$\begin{aligned} w(0) - w(-s) &= u(x,t) - u(x-c_1 t, 0) \\ &= u(x,t) - u_0(x-c_1 t) \end{aligned} \tag{51}$$

so we have the solution:

$$u(x,t) = u_0(x-c_1 t) + \int_{0}^{t} \alpha\big(v(x+c_1(s-t), s) - u(x+c_1(s-t), s)\big) ds. \tag{52}$$

Similarly we can do for the equation (47), hence we have

$$v(x,t) = v_0(x-c_2 t) + \int_{0}^{t} \alpha\big(u(x+c_2(s-t), s) - v(x+c_2(s-t), s)\big) ds. \tag{53}$$

Another way we can treat two channels dissipation model: first by transformations and then by characteristic method. Transform $u(x,t)$ and $v(x,t)$ by

$$\begin{aligned} u(x,t) &= e^{-\alpha t} U(x,t) \\ v(x,t) &= e^{-\alpha t} V(x,t) \end{aligned} \tag{54}$$

so we have new initial value problem:

$$\begin{aligned} & U_t(x,t) + c_1 U_x(x,t) = \alpha V(x,t) \\ & V_t(x,t) + c_2 V_x(x,t) = \alpha U(x,t) \\ & U(x,0) = u_0(x) \\ & V(x,0) = v_0(x) \end{aligned} \tag{55}$$

The equations (53) are solved by characteristic method, we obtain:

$$U(x,t) = u_0(x-c_1 t) + \int_{0}^{t} \alpha V(x+c_1(s-t), s) ds \tag{56}$$

$$V(x,t) = v_0(x-c_1 t) + \int_{0}^{t} \alpha U(x+c_2(s-t), s) ds \tag{57}$$

or

$$u(x,t) = u_0(x-c_1 t) e^{-\alpha t} + \int_{0}^{t} \alpha v(x+c_1(s-t), s) e^{\alpha(s-t)} ds \tag{58}$$

$$v(x,t) = v_0(x-c_2 t) e^{-\alpha t} + \int_{0}^{t} \alpha u(x+c_2(s-t), s) e^{\alpha(s-t)} ds . \tag{59}$$

The equation (52)-(53) and (58)-(59) are called integral equation form of the initial value of the two channel dissipation model. It is very necessary to prove the existence solution of the initial value problem of two channel dissipation model by Fixed Point Theorem.

4. CONCLUSION

The particular solutions of this model can be obtained by using the elimination method and canonical transformation. The model becomes Telegrapher or Klein Gordon equation in the second order partial differential equation. In the Klein Gordon equation we get particular solutions, so we also obtain particular solutions of the linear two channels dissipation model. In the form Telegrapher equation we solve by Fourier transform and obtain the solution of the linear two channels dissipation model in the Fourier representation formula. Finally we find the analytical solution by characteristic method in the integral form of initial value problem of the linear two channels dissipation model. For further work we will try to analyze the model by Semigrup Operator and look for an analytic solution with piecewise constants velocities.

Acknowledgement

For writing this paper, we would like to thank to dr. Frits van Beckum who gave us this problem, as preparing for nonlinear case, when he visited at Department of Mathematics, Gadjah Mada University and his nice communications up to now for doing our research. Research was partially completed while the first author was visiting the Institute for Mathematical Sciences, National University of Singapore in 2006 on the Workshop Geophysical Fluid Dynamics and Scalar

Transport in the Tropics(13 Nov- 8 Dec 2006). We also thank all the anonymous for reading the paper carefully.

REFERENCES

Evans, L.C., 1998, *Partial Differential Equations*, American Mathematical Sociaty

Gupalo, Y. P., Polyanin, A. D., Ryanzantsev Y.S., *Mass and Heat Exchange between Reacting Particles and Flow*, Nauka, Moscow

Mascia, C., Natalini, R. 1996, L^1 Nonlinear Stability of Travelling Waves for Hyperbolic system with relaxation, *Journal of Differential Equations*, 132, 275-292

Pinsky M.A., 1998, *Partial Differential Equations and Boundary Value Problems with Applications*, McGraw-Hill Companies Inc.

Polyanin A.D., 2002, *Handbooks of Linear Partial Differential Equations for Engineers and Scientists*, Chapman & Hall/CRC

Sumardi, Soeparna D. Lina Aryati, 2005 Convergence of finite difference approximation for two channel dissipation model, *Proceeding International Conference on Applied Mathematic 2005 (ICAMO5)*

Sumardi, Soeparna D. Lina Aryati, 2007, The immersed interface method for two channels dissipation model with discontinue constants velocity, *Proceeding of SEAMS-GMU Conference 2007*

Van Beckum, F. P.H., Travelling wave solution of a coastal zone non-Fourier dissipation model, 2003, *Proceeding in Symposium on Coastal Zone Management,* ITB, Bandung