Application of non linear models in estimating growth curves of body weight and sizes of Holstein-Friesian female cattle

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ABSTRACT: Aplication of non-linear mathematical models in estimating growth curves of Holstein-Friesian (HF) females were tested. Body weight and measurements were conducted for various ages of HF females, succesively for calves 54 hd, heifers 84 hd and adults 348 hds in a west area of KPSBU Lembang, West Java in July-August 2007. Body weight was converted from chest girth using a Gompertz tape. Growths of body weight and sizes generally formed sigmoid pattern, increased rapidly from birth, reached maximal around 9.58 mo (an estimated pubertal age), then slowed to an almost constant at the age of ≥52 mo. Three non-linear models used in estimating growth curbes were logistic, von Bertalanffy and Gompertz respectively. Comparison of the best model was determined for the highest determinant coefficient (R^2) and the lowest mean square of error (MSE). Gompertz and von Bertalanffy models were considered as the two most appropiate models due to their almost similar values in R^2 and MSE. The easiness in developing model measured by counting lowest iteration number was succesively for von Bertalanffy, Gompertz, and Logistic. Logistic model predicted better in biological aspects as its closer predicting pubertal age (around 9.58 mo), but the two remaining were better from the mathematical aspects due to easier and more suitable uses.

Key words: HF females, body weight, body measurement, growth curve, non-linear model

INTRODUCTION

Improving dairy cows for highly milk ability requires replacement stocks of heifers with good growth rates. Normal growth pattern of individual animal or population usually forms a sigmoid pattern as well known the shape of letter S. Sigmoid curve describes the pattern of acceleration and deceleration of growth (Brody, 1945). Growth pattern as a simple form with the highest growth rate occurres in early life, increases slowly until reaching a constant at a mature age, then slowly decrease (Lawrence and Fowler, 2002). When the growth reaches a maximum speed, this is expressed as an inflection point. Brody (1945) and Campbell *et al*. (2003) describes an inflection point occurs when the growth reaches a maximum indicating that the change from increased acceleration to deceleration of growth. The inflection point also reflects the age of puberty, the lowest probability of death, and the geometric reference for the determination of the age between animals or populations.

Puberty or sexual maturity is the age of which the reproductive organs begin to function for the development and influence of pituitary hormones on body growth, uterus, ovarian and other reproductive tracts (Toelihere, 1994). It is importance to consider an early pubertal age be achieved by the heifers at an expected body weight as well as for an enough size, so that she can calving safely at the first calving. An early age at first calving would produce slightly lower milk production in first lactation of heifers, but have the advantage of a longer productive lifetime than those of slower age first calving heifers (Salisbury and Van Demark, 1985).

As previously described, growth curve normally is described as a sigmoid shape, but the growth curve in fact varies between animals or populations. Growth curve is necessary be estimated to know the real growth pattern from a population, as the growth is determined by the resultant between genetic and environmental components. Accurate estimation of the growth curve can be used as a standard curve pattern of livestock population, and to identified performance individual heifer and cow as replacement stocks. The estimation of growth curve by non linier mathematic models such as Logistic, von Bertalanffy and Gompertz were already used to estimate the biological values of growth curve in sheep, beef cattle and dairy cattle (Suparyanto *et al*., 2004; Mauluddin, 2005; Sengül dan Kiraz, 2005; Akbas *et al.*, 2006).

This study was aimed to estimate growth curve in terms of the changes in body size and body weight based on the additional ages of HF females in a western part KPSBU Lembang, West Java using those three mathematical non linier models.

MATERIALS AND METHODS

Location

This research was conducted in a west part region of KPSBU West Java, in Lembang Distrcit, West Java Province from July 16 to August 23, 2007. Research locations were in two villages of Sukajaya and Cikahuripan consisting of eight milk collecting locations. Body measurements were taken at randomly selected cattle from each milk collecting location about 10-20%.

Materials

Holstein Friesian (HF) females were used for a total number of animals 532 hds but those of 46 hds were removed for outliers and less complete. After data reduction, total data of HF females were used 486 hds consisting of calf at the ages of 0-8 mo. for 54 hds, heifers 8-24 mo. for 84 hds and adult over 24 mo. for 348 hds. Measuring tape and yard stick were used for collecting data.

Prediction of the ages was by interview to farmers, and based on information of changing incisors. Various body measurements were taken for: shoulder hight (SH) from the highest starting point shoulder (thoracic vertebrae os) vertically to the surface of the soil, body length (BL) from the edge of the humerus bone to bone sitting (tube ischii), chest circumference (CC) from wrapping around the chest behind the shoulder joint (os scapula), chest width (CW) ranging from the distance between the left and right shoulder joint, chest depth (CD) measured from the highest point of the shoulder (thoracic vertebrae, os) to sternum (os sternum) lower back of the front legs.

All of those body measurements were measured by measuring stick at a centimeter (cm) unit, whereas body weight (kg) converted from tapes Gordas NASCO based on chest circumference.

Growth curves were predicted by three mathematical non linier models, as followed:

a. Logistics: $Yt = A * (1 + B * e (-k * t)) -1$
b. Gompertz: $Yt = A * \exp (-B * e (-k * t))$
c. von Bertalanffy: $Yt = A * (1-B * e (-k * t))3$

where:

A: body weight (kg) or body size (cm) Adult (asymptotes), namely the value of t approaches infinity;
B: scale parameter (integral constant value);
e: logarithmic base (2.71828);
k: the average rate of growth until cattle reaching adult body;
Yt: body weight (kg) of body size (cm) of cattle at time t; and
t: age of cattle (in months).

Statistical calculations were processed by package program SAS ver 6:12 with Marquardt procedure. Coefficient values were used to stimulate the alleged changes in body size in accordance with the changes in desired times. The number of iterations were used to estimate the easiness level of three mathematical models. Marquardt procedure had to be decreased with A, B and k parameters to achieve convergence, so that it could be predicted the values of A, B and k. Derivative equations in getting A, B and k parameters from each mathematical model condidered was presented in Table 1.

Derivating t was used to find the velocity of growth and acceleration of growth through firstly derivating on growth rate (dY/dt). Meanwhile, the acceleration of growth through secondly derivating growth rate (d2Y/dt2), so the acceleration of growth was zero (d2Y/dt2 = 0) was an inflection point with maximum velocity.

Table 1. Derivative equation in getting A, B, k and t parameters from three mathematical models

Model Matematik	dY/dA	dY/dB	dY/dk
Logistic	$(1+B*e^{(-k*t)})^{-1}$	$(-A)*e^{(-k*t)}*(1+B*e^{(-k*t)})^{-2}$	$A*B*t*e^{(-k*t)}*(1+B*e^{(-k*t)})^{-2}$
von Bertalanffy	$(1+B*e^{(-k*t)})^{-1}$	$(-3A)*e^{(-k*t)}*(1+B*e^{(-k*t)})^{2}$	$3*A*B*t*e^{(-k*t)}*(1-B*e^{(-k*t)})^{2}$
Gompertz	$\exp(-B*e^{(-k*t)})$	$(-A)*e^{(-k*t)}*\exp(-B*e^{(-k*t)})$	$A*B*t*e^{(-k*t)}*\exp(-B*e^{(-k*t)})$
	dY/dt	d^2Y/dt^2	
Logistic	$A*B*k*e^{(-k*t)}(1+B*e^{(-k*t)})^{-2}$	$A*B*k^2*e^{(-k*t)}(1+B*e^{(-k*t)})^{-3}(B*e^{(-k*t)}-1)$	
von Bertalanffy	$3*A*B*k*e^{(-k*t)}*(1-B*e^{(-k*t)})^{2}$	$(-3)*A*B*k^2*e^{(-k*t)}*(1-4*B*e^{(-k*t)}+3*B^2*e^{(-2*k*t)})$	
Gompertz	$A*B*k*e^{(-k*t)}*\exp(-B*e^{(-k*t)})$	$A*B*k^2*e^{(-k*t)}*\exp(-B*e^{(-k*t)})*(B*e^{(-k*t)}-1)$	

A: body weight (kg) or body size (cm) of adult (asymptotes), namely the value of t approaches infinity; B: scale parameter (integral constant value);e: logarithmic base (2.71828); k: average rate of growth until the cattle reach maturity; Yt: body weight (kg) of body size (cm) of cattle at time t; andt: age of cattle (in months).

RESULTS AND DISCUSSION

General Condition of Research Location

Dairy regions of small dairy farmers under the supervision of KPSBU Lembang were generally divided into three working areas, western, central and eastern working area. Western working part consisted of eight Milk Collecting Unit (MCU), successively Manoko, Karamat, Pasar Kemis, Nagrak, Corner, Pamecelan, Citespong and Barunagri. All there MCUs were located in two villages of Sukajaya and Cikahuripan. Physical conditions for rainfall 2200 mm/yr, total rainy period 8 mo., average of temperature 19-22 ^{o}C, altitude 1200 m asl and hilly landscape.

Description of Growth Pattern

Table 2 presented real data of the descriptions on body sizes and weight at various ages of Holstein Friesian (HF) females in filed research of KPSBU Lembang. Growth pattern of HF dairy heifers and cows in the western part of KPSBU Lembang in term of the changing body weights by the adding ages generally showed a sigmoid pattern. Since birth or the age of less than one month experienced a large increase in growth velocity, reached maximum growth at an inflection point approximately 9.58 mo. of age of puberty, then experienced slow growth in an increasingly slower pace until it reached an almost constant growth in more than 52 mo of age.

Table 2 showed that HF female calves at the 0-2 mo ages had weights around 43- 99 kg and shoulder height around 79-105 cm. According to Salisbury and VanDemark (1985), birth weight and size of HF calves in New York (USA) was 46.5 kg and shoulder height 75.14 cm meaning that HF calves in this study were lower than those in New York. In contrast, birth weight of HF females in dairy station at BPTU Baturraden was 39.99 kg (Anggraeni, 2006) suggesting birth weights of HF females in this study though raised semi intensively by small holders still better than those raised in an intensive management of dairy station.

Puberty predictively occurred at around 5-15 mo. for the growths of body weightl while those for chest circumference and body length around 295.5±49,3 kg, 155,3±9,8 cm, and 112,1±12,7 cm. These were still closer to the growth ranges of HF in New York for body weight 279.4± 34.9 kg, chest circumference 149.1±7.4 cm, and body length 116.6±5.3 cm respectively (VanDemark and Salisbury, 1985). A prior study by Akbas *et al*. (2006) reported pubertal age and body weight for HF male were 8.2 mo and 274 kg. The observable pubertal age of HF heifers in this study was estimated 9-12 mo. The first mating domestic HF was suggested at the age 15 mo with an expected body weight 275 kg (Sudono, *et al*., 2003). At the range ages of 11-14 mo, average body weights of HF in this study was

Table 2. Statistical description of various body sizes and weight of Holstein Friesian at various ages

Age (Month)	Sample (hds)	Variable	DD	LeD	PB	TP	LD	BW (kg)
			(cm)					
0-2	19	$\bar{X}$ ± SB	32,6±4,3	19,4±3,9	71,6±13,9	80,2±6,9	89,4±8,6	63,0±18,2
		Min-Mak	24,5-40,0	8,0-27,0	24,0-88,0	70,0-96,0	79,0-105,0	43,0-99,0
3-4	15	$\bar{X}$ ± SB	40,8±4,3	24,2±2,4	89,0±11,4	93,5±10,3	107,7±11,3	108,2±33,8
		Min-Mak	34,0-50,5	20,5-29,0	69,5-111,0	77,0-112,0	91,0-132,0	65,0-188,0
5-6	9	$\bar{X}$ ± SB	45,5±3,9	25,4±3,2	99,9±9,8	98,6±11,5	121,0±11,8	149,4±43,3
		Min-Mak	40,0-52,0	22,0-30,5	85,0-116,0	74,5-114,0	107,0-145,0	102,0-242,0
7-8	11	$\bar{X}$ ± SB	49,9±3,6	27,7±2,9	104,6±8,3	107,0±5,4	130,0±7,9	180,9±32,5
		Min-Mak	45,0-58,0	22,0-32,0	87,0-118,0	100,5-119,0	121,0-149,0	146,0-262,0
9-10	4	$\bar{X}$ ± SB	53,8±2,7	29,0±2,2	112,1±12,7	111,9±1,3	140,3±7,0	200,5±37,7
		Min-Mak	50,0-56,5	26,0-31,0	99,0-128,0	110,0-113,0	132,0-147,0	158,0-248,0
11-12	10	$\bar{X}$ ± SB	58,8±2,6	32,6±2,9	127,7±8,3	118,9±4,6	155,3±9,8	295,5±49,3
		Min-Mak	55,5-62,0	28,0-37,0	114,0-138,5	114,0-129,5	140,0-167,0	220,0-356,0
13-14	6	$\bar{X}$ ± SB	60,6±1,0	33,8±1,2	127,1±4,1	119,5±2,3	156,7±4,8	301,2±25,1
		Min-Mak	59,0-62,0	33,0-36,0	121,0-131,0	117,0-122,0	151,0-163,0	272,0-335,0
15-16	5	$\bar{X}$ ± SB	61,5±5,8	35,2±6,2	129,2±8,6	122,2±8,0	164,8±13,9	346,6±77,8
		Min-Mak	55,5-69,0	28,0-45,0	122,0-143,0	114,5-131,0	153,0-184,0	282,0-457,0
17-18	11	$\bar{X}$ ± SB	62,7±3,6	35,4±3,2	137,2±8,0	122,8±3,6	170,0±10,2	374,5±58,8
		Min-Mak	59,5-70,0	31,5-43,0	123,0-153,0	117,5-129,0	158,0-187,0	308,0-477,0
19-20	4	$\bar{X}$ ± SB	66,5±2,7	38,9±3,9	146,4±2,2	128,6±4,9	180,5±6,0	435,5±38,0
		Min-Mak	64,0-70,0	33,5-42,5	144,0-148,5	123,0-134,0	174,0-188,0	395,0-484,0
21-22	4	$\bar{X}$ ± SB	64,9±3,8	36,8±1,7	144,3±8,8	127,8±6,7	171,3±15,3	384±89,2
		Min-Mak	62,0-70,5	35,0-39,0	133,0-153,0	118,0-133,0	156,0-190,0	297,0-496,0
23-24	40	$\bar{X}$ ± SB	67,7±2,9	38,1±3,2	144,8±8,0	130,2±3,7	177,3±8,0	416,2±48,5
		Min-Mak	60,5-75,0	32,0-45,5	129,5-160,0	122,0-139,0	157,0-197,0	303,0-545,0
25-30	25	$\bar{X}$ ± SB	67,7±3,8	38,3±3,6	149,3±7,8	128,7±3,8	177,9±8,6	420,4±52,5
		Min-Mak	61,0-79,0	32,5-44,0	135,0-161,0	120,0-141,0	161,0-194,0	324,0-523,0
31-36	62	$\bar{X}$ ± SB	69,1±3,2	38,7±3,3	150,6±7,3	130,8±5,0	179,3±8,8	428,5±54,9
		Min-Mak	62,0-77,0	31,0-48,0	135,0-168,0	120,0-144,5	161,0-201,0	324,0-572,0
37-42	26	$\bar{X}$ ± SB	69,2±2,7	39,5±3,5	150,2±7,0	130,9±3,5	180,0±7,2	432,0±45,7
		Min-Mak	63,0-73,0	34,0-49,0	136,0-166,0	125,0-139,0	166,0-198,0	352,0-551,0
43-48	83	$\bar{X}$ ± SB	70,0±2,7	40,6±4,3	154,2±7,7	130,5±4,5	182,6±8,0	448,8±50,9
		Min-Mak	63,0-78,0	31,0-54,0	135,0-173,5	120,0-143,0	164,0-201,0	339,0-572,0
49-54	20	$\bar{X}$ ± SB	70,4±4,1	41,0±4,1	153,7±6,3	130,6±5,1	184,2±8,9	459,7±58,2
		Min-Mak	63,0-80,5	35,0-49,0	143,0-164,0	123,0-139,0	170,0-203,0	370,0-585,0
55-60	54	$\bar{X}$ ± SB	70,6±3,0	41,2±4,0	157,2±6,5	131,7±4,5	183,7±7,5	455,5±48,4
		Min-Mak	65,0-81,0	31,0-50,0	141,5-173,5	123,0-143,5	167,0-192,0	356,0-572,0
61-66	12	$\bar{X}$ ± SB	71,1±1,7	42,0±2,6	153,2±5,6	130,5±5,8	184±4,95	457,2±32,4
		Min-Mak	69,0-74,0	37,0-46,0	143,0-163,5	119,0-141,0	177,0-198,0	413,0-510,0
67-72	31	$\bar{X}$ ± SB	70,5±2,9	41,1±3,5	157,2±6,1	131,0±4,5	182,5±5,8	448,1±38,0
		Min-Mak	65,0-80,0	35,0-49,5	145,0-175,0	122,0-141,0	175,0-197,0	400,0-545,0
73-78	2	$\bar{X}$ ± SB	74,5±0,7	47,5±0,7	162±4,2	133±7,1	193,5±0,7	520±4,2
		Min-Mak	74,0-75,0	47,0-48,0	159,0-165,0	128,0-138,0	193,0-194,0	517,0-523,0
79-84	12	$\bar{X}$ ± SB	72,3±1,6	45,3±3,5	159,6±7,1	131,9±3,1	188,5±5,5	486,4±36,9

Table 2 (Continued). Statistical description of various body sizes and weight of Holstein Friesian at various ages

Age (Month)	Sample (hds)	Variable	DD	LeD	PB	TP	LD	BW (kg)
		Min-Mak	69,0-74,5	40,0-51,0	148,0-173,0	126,0-137,5	181,0-198,0	436,0-551,0
85-90	3	$\overline{X}$ ± SB	73,0±3,6	41,5±0,9	157±1,7	133,7±2,1	188±8,7	484±58,4
		Min-Mak	69,0-76,0	40,5-42,0	156,0-159,0	132,0-136,0	182,0-198,0	444,0-551,0
91-96	4	$\overline{X}$ ± SB	71,6±1,7	41,9±5,8	154,3±5,3	132,75±3,4	188,3±6,9	485,5±45,2
		Min-Mak	70,0-74,0	33,5-46,0	150,0-162,0	128,0-136,0	179,0-194,0	425,0-523,0
97-108	3	$\overline{X}$ ± SB	71,8±1,8	47,3±4,6	158±11,3	127,5±2,1	184±1,4	456,5±7,8
		Min-Mak	70,5-73,0	44,0-50,5	150,0-166,0	126,0-129,0	183,0-185,0	451,0-462,0
120	7	$\overline{X}$ ± SB	70,9±2,2	46,4±6,9	159,3±7,2	131,5±3,8	184,6±5,2	461,2±34,0
		Min-Mak	68,0-74,0	37,5-56,0	150,0-170,0	126,0-137,0	176,0-191,0	406,0-503,0

295.5±49.3kg to 301.2±25.1 kg, indicating that these heifers reaching puberty faster than the prior recommendation.

Table 2 also showed in general the growths slightly increased after the age of more 24 month as indicated by decreasing body weight and sizes by the additional ages of HF females. Deresz *et al*. (1987) recommend a good body condition was required during calving and required adequate nutrition supply during early lactation to obtain high production in cows. Presented by bisual information of first calving occurred at the age ranges of 24.4-30.4 mo. it was be able to disseminate the information of allowing first mating age around 11-14 mo. , so the first calving was able be reached early ages around 20-23 mo.

Estimation of Growth Curve

Estimated growth curve parameters of A, B and k of the three non linier mathematical models were presented in Table 3. The value of A indicated the maximum value of body size (cm) or body weight (kg), oftenly known as asymptotes to maturity, B showed the scale parameter or the integral constants, and K indicated the average growth rate until mature body weight reached. Choosing three mathematical models of Gompertz, Von Bertalanffy and Logistic was considered for the easiness of calculation processes, especially in making derivatives of a function with only three parameters (A, B and k).

The asymptotes (A) having the largest value were succesively for von Bertalanffy, Gompertz and Logistic models (Table 3). Nevertheless asymptotes among the three models showed no sufficient difference. The asymptotes (A) of body weight, chest circumference, chest width, body length and shoulder height were succesively 448.0- 453.5 kg; 70.2-70.4 cm; 182.7-183.3 cm, 41.3- 41.6 cm, 154.9-155.6 cm and 130.7-130.8 cm. The differences of B and k values showed the difference between mathematical functions. In general, B and k values of Logistic were higher than those of von Bertalanffy and Gompertz. Standard error (SE) showed that the smaller SE value of A, B and k parameters indicated the uniformity of the these values.

By plotting the estimated growth curves by the three models (not presented here) illustrated in general that the velocity phase gradually grew from birth until an inflection point at the age of 17-20 mo, while achieving constant growth around the age of 42-60 mo. For chest hight and shoulder height, both entered the slowly growth rate at the beginning inflection point until the age of 17-18 mo. Body weight, chest circumference, chest width and body length reached velocity phase slowly at the beginning inflection point until the age of 20 mo. Gompertz achieving constant growths in body weight, chest deth, body length and chest circumference were around the age of 42 mo. Von Bertalanffy achieving constant growth in body weight, chest depth, chest circumference, chest width, body length and shoulder height were about the ages of 42, 42, 46, 54, 42, 56 mo. respectively. Further, Logistic achieving constant growth on body weight, chest depth, chest circumference, chest width, body length and shoulder height were about at the ages of 42, 42, 36, 42, 46 and 36 mo. respectively.

Table 3. The values of A, B and k of body sizes and weight of Holstein-Friesian females estimated from von Bertalanffy, Gompertz and Logistic models

Variable	Model	A ± SE	B ± SE	k ± SE	Equation
	Bertalanffy	453.5±3.0	0.59±0,03	0.12±0.00	$454*(1-0{,}59*e^{(-0{,}118*t)})^3$
BW	Gompertz	451.6±2.9	2.40± 0.15	0.14±0.00	$452*Exp(-2{,}40*e^{(-0{,}136*t)})$
	Logistic	448.0±2.7	6.49± 0.72	0.20±0.01	$448*(1+6{,}49*e^{(-0{,}195*t)})^{-1}$
	Bertalanffy	70.4±00.2	0.26±0.01	0.12±0.00	$70*(1-0{,}26*e^{(-0{,}117*t)})^3$
CD	Gompertz	70.4± 0.2	0.89±0.02	0.13±0.00	$70*Exp(-0{,}89*e^{(-0{,}125*t)})$
	Logistic	70.2± 0.2	1.33±0.05	0.15±0.00	$70*(1+1{,}32*e^{(-0{,}150*t)})^{-1}$
	Bertalanffy	183.3± 0.5	0.25±0.01	0.12±0.00	$183*(1-0{,}25*e^{(-0{,}122*t)})^3$
CC	Gompertz	183.1± 0.5	0.86±0.02	0.13±0.00	$183*Exp(-0{,}86*e^{(-0{,}130*t)})$
	Logistic	182.7± 0.5	1.27± 0.05	0.16±0.00	$183*(1+1{,}27*e^{(-0{,}156*t)})^{-1}$
	Bertalanffy	41.6± 0.3	0.23±0.01	0.08±0.00	$42*(1-0{,}23*e^{(-0{,}0794*t)})^3$
CW	Gompertz	41.5± 0.3	0.78±0.04	0.09±0.00	$42*Exp(-0{,}78*e^{(-0{,}085*t)})$
	Logistic	41.3±0.3	1.11± 0.07	0,10±0.00	$41*(1+1{,}11*e^{(-0{,}103*t)})^{-1}$
	Bertalanffy	155.6±0.5	0.25±0.00	0.10±0.00	$156*(1-0{,}25*e^{(-0{,}101*t)})^3$
BL	Gompertz	155.4±0.5	0.84±0.02	0.11±0.00	$155*Exp(-0{,}86*e^{(-0{,}108*t)})$
	Logistic	154.9±0.5	1026±0.05	0.13±0.01	$155*(1+1{,}26*e^{(-0{,}129*t)})^{-1}$
	Bertalanffy	130.8±0.4	0018±0.00	0.14±0.01	$131*(1-0{,}18*e^{(-0{,}140*t)})^3$
SH	Gompertz	13008±0.4	0060±0.02	0.15±0.01	$130*Exp(-0{,}60*e^{(-0{,}147*t)})$
	Logistic	130.7±0.4	0078±0.04	0.17±0.01	$131*(1+0{,}78*e^{(-0{,}166*t)})^{-1}$

A: body weight (kg) or body size (cm) at mature age (asymptotes), namely on the t value approaching infinity;
B: scale parameter (integral constant value);
K: average growth rate until reaching body maturity;
SE: standard error,
BW: Body weight, CC: circumference chest, BL : body length, HS: shoulder height, CW: chest width, DC: chest depth.

Growth Velocity

The speed of growth or dy / dt or YTL was gained from the first derivative of each mathematical model (Yt). Growth velocity when t was zero or when the livestock were born were obtained for body weight, chest circumference, shoulder height, body length, chest width and chest depth succesively 10.1-15.9 kg/m, 7.0-9.5 cm/mo, 5.3- 6.7 cm/mo, 4.9-6.7 cm/m, 1.1-1.4 cm/mo and 2.9-3.5 cm/mo. The growth velocity since the birth having the values from the highest to the lowest were succesively for von Bertalanffy, Gompertz and Logistic.

Inflection point when the growth rate obtained for body weight, chest circumference, shoulder height, body length, chest width and in the chest were succesively 21.9-23.8 kg/mo, 7.1-10.0 cm/mo, 5.4-8.2 cm/mo, 5.0-7.0 cm/mo, 1.1-1.5 cm/mo and 2.6-3.7 cm/mo. At the inflection point, body size was negative, so the growth speed since at birth indicated the decreases. Unlike that of body weight with a positive inflection point for all of those mathematical models reached the maximum point because of smaller growth rate at birth than the inflection point.

That was in accordance with the statement Brody (1945) explaining that an inflection point as the maximum growth, the pubertal age and the lowest mortality rate. Growth velocity during the age of 60 mo. indicated the decreasing growth rate of overall body measurements and body weight compared to those at birth or at an inflection point due to the oldness.

Accelerating Growth

Growth acceleration or d2y/dt2 or Ytll indicated for cattle experiencing growth velocity by the times in order to get positive and negative values. Positive value meant that cattle experienced a rapid growth and vice versa.The growth acceleration values obtained showed that body weight had a positive age at birth, meaning that a calve was still in the stage of rapid growth. In contrast, body sizes had negative growth acceleration value at birth meaning that livestock was already experiencing slower growth. Acceleration of zero indicated cattle reaching an inflection point because the time acceleration of growth was zero then the maximum value of growth velocity. Brody (1945) explained that the inflection point indicated the time to reach maximum growth (shift from increasing to decreasing growth rate).

Based on the point of inflection and speed of growth discussed previously, the growth acceleration was zero point on the pubertal body weight. When reaching the age of 60 mo., the animals reached the constancy. Age began reaching a slower growth from the estimating of von Bertalanffy for chest width, chest depth, body length, shoulder height, chest circumference and body weight at the ages respectively at 6, 6, 6, 2.5, 5; and 13.5 mo; while those of Gompertz at the ages of 8, 7, 8, 3, 7, and 12 mo, then those of Logistic at the ages of 14, 10, 12; 6.5, 10, and 17 mo. respectively.

All these results pointed that cattle experienced a slowing growth then reaching constant allowing maturity. Body weight at age 52-180 mo reached a constant growth, while those of body sizes were over the age of 62 mo in the Gompertz model, 75 mo in von Bertalanffy and 66 mo in Logistic.

Inflection Point

Inflection point represents a growth phase to achieve maximum growth rate and a zero acceleration. Brody (1945) explained that the inflection point indicates achieving maximum growth or displacement of an increase to a decrease growth rate. This inflection point indicates that the animal experiences puberty, so that an indicator of animal productivity when viewed from the aspects of reproduction. Puberty estimated by the three models was at the age ranges of 4.8-9.6 mo. Salisbury and VanDemark (1985) stated the average pubertal age in cattle feed in a normal condition was 9 mo or around 5-15 mo.

Holstein-Friesian heifer was recommended getting the first mating at the age of 15 mo and body weight at 275 kg (Sudono *et a*l., 2003). Holstein-Friesian heifers in this study had body weight at the ranges of 295.5± 49.3 to 301.2±25.1 kg and ages of 11-14 mo, so these results indicated heifers in this study achieving a faster growth. Attaining pubertal age of 9.58 mo by Logistic could be a reference for mating time of HF heifers raised by small holders in KPSBU Lembang at the range ages of 11-14 mo. And body weights of 246.2-326.3 kg.

Suitability and Easiness of Growth Curve Mathematical Models

Investigation on the suitability of the three mathematical models to estimate the growth curve of HF females could be searched by calculating the determinant coefficient (r^2) and the mean square error (MSE). Mathematical models having the highest r^2 and the lowest MSE was considered as the most suitable model (Maharani *et al*., 2001). Determinant coefficient (r^2) indicates a statistical relationship of changes in body size and body weight with age of which a higher r^2 value indicating a

closer statistical relationship. Mean square error (MSE) showed the sum of the squares of the difference between the actual value with the expected value with a smaller MSE indicating a greater suitability of the allegations with actual values.

The smallest r^2 and the lowest MSE in Table 4 showed the most suitable model with the actual data and then ranked from the smallest to the largest. The smallest rangking MSE generally showed the smallest MSE, by contrast, the the largest ranking r^2 generally showed the greatest. Based on Table 4, the smallest MSE and r^2 were identified succesively for von Bertalanffy, Gompertz, and Logistic.

Table 4. The suitability of mathematical models to estimate growth curve of Holstein-Friesian females

Variabel	von Bertalanffy		Gompertz		Logistic	
	MSE	r^2	MSE	r^2	MSE	r^2
BW	2354,53 (2)	83,30 (2)	2353,90 (1)	83,30 (3)	2388,08 (3)	83,06 (1)
CD	9,55 (1)	90,68 (3)	9,58 (2)	90,65 (2)	9,76 (3)	90,48 (1)
CC	65,72 (2)	89,90 (2)	65,61 (1)	89,92 (3)	65,77 (3)	89,89 (1)
CW	14,49 (1)	69,21 (2)	14,56 (2)	69,07 (1)	14,76 (3)	68,63 (3)
BL	61,00 (1)	88,51 (3)	61,44 (2)	88,43 (2)	63,00 (3)	88,14 (1)
SH	44,41 (3)	78,05 (1)	44,37 (2)	78,07 (2)	44,36 (1)	78,07 (3)
Total	(10)	(13)	(10)	(13)	(16)	(10)

Horizontal values in parentheses indicated the rank of MSE and r^2 from the smallest to the largest
BW: Body weight, CC: chest circumference, BL: body length, SH: shoulders height, chest width: CW, CC: chest depth.

The easiness of a model can be done by knowing the number of iterations. As stated by Suparyanto (1999) iterative processed recovering the sum of squares from the analysis of non-linear least squares to achieve convergence of each parameter. The smallest number of iterations had the greatest level of convenience indicating the requirement of achievinf a faster convergence. Based on this calculation, the level of convenience ordered from the largest mathematical models were succesively for von Bertalanffy, Gompertz, and Logistic.

Based on comparision on these three mathematical models, it was recommended to be better of applying von Bertalanffy model to get the most suitability and the most easiness in estimating HF females in the western part of KPSBU Lembang. However, from the biological aspect, Logistic was recommended to estimate better growth curve of these HF females because of its advantage in estimating the expected pubertal age approximately at 9.58 mo.

CONCLUSIONS

Growth curve of HF females in the western part of Lembang KPSBU showed a sigmoid pattern showing that since the birth (less than one month of age) experienced a large increase in the growth, reached a maximum velocity growth at approximately 9.58 mo. of age, then reached a slow growth and almost constant at around 52 mo, of age. By using three non-linear mathematical models to estimate growth curve of HF females showed that Gompertz and von Bertalanffy were considered as the two most appropriate models due to their almost similar values in R2 and MSE. However the easiness in developing growth curve as indicated by the lowest iteration number was succesively for von Bertalanffy, Gompertz, and Logistic. Logistic model was identified as the best model in predicting growth curve in tem of biological aspect because of it was closer in predicting the age of pubertal age (around 9.58 mo), but the two remaining models were better from the mathematical aspects because they were easier and more suitable be used for growth estimation.

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