FITTING OF GOMPERTZ CURVE FOR GROWTH OF SHEEP IN SARAWAK, MALAYSIA.

S.Abdul Wahid[1] and J.M.Yunus[2]

ABSTRACT

An experiment was conducted using randomized complete block design to study the growth of crossbred sheep at two locations in Sarawak, Malaysia. The two sites were one in Samarahan and the other in Bau, which held 348 and 73 sheep respectively. Data on birth weight and subsequent body weights were collected every 28 days to 364 days of age. The scattered plot of body weight of sheep against time exhibited approximately a sigmoid shape. A Gompertz model was fitted on happenstance body weight data using SAS package (PROC NLIN) on mainframe computer at MARDI, Serdang. Ad hoc starting values of the model parameters helped the Levenburg-Marquardt iterative algorithm to globally converge to stationary points in 12 and 7 iterations for model parameter estimations of body weight at Samarahan and Bau experimental sites respectively. The fitted models were diagnosed using nonlinear model adequacy measures. On examination of the residual scatter plots, the Gompertz curve exhibited apt function to describe sheep body weight. The relative growth rates (RGR) at Samarahan and Bau were 0.0983 and 0.0962 kg kg^{-1} $month^{-1}$ respectively.

Key words: Gompertz curve, Growth, Body weight, Sheep, Sarawak, Malaysia

INTRODUCTION

For a long time growth functions have been used to provide a quantitative description of time-oriented observations on the growth of organism or its part. The use of growth functions is empirical. The established growth functions have some biological inferences whose parameters are characteristic to some underlying physiology or biochemistry of growth. In 1825, Gompertz and later followed by Bliss (1970), Couston and Venus (1981), Ratkowsky (1983) and Seber and Wild (1989), proposed an asymmetrical sigmoid curve that would be expressive of the law of human mortality. The Gompertz curve having three parameters arranged as a double exponent could be written as:

$$f(x) = a\,e^{-e^{B-Yx}}$$

where a , B and Y are parameters of the nonlinear function. a represents maximum or minimum growth of the animal, B is a curvature constant and Y is the rate parameter which is the absolute rate of growth of the animal.

Several measurements of size such as weight, length, height and circumference may give rise to growth curves of the same shape. The growth rate expressible in differentiable equation is:

$$\frac{df}{dx} = Y\,f\,(\log a - \log f)$$

where $Y > 0$ and $a > 0$. And, the relative growth rate (RGR) of Seber and Wild, (1989) is estimated using the following equation:

$$RGR = \frac{\log f_{i+1} - \log f_i}{X_{i+1} - X_i}$$

Where f is the function and X is the independent variable or time period.

The growth function was initially devised for use in quantitative analysis of animal growth where Medawar (1940) had derived the Gompertz model for the heart of a chicken. The Gompertz model had also been

[1] B-27-Taman Bukit Kajang, Jalan Bukit, 43000 Kajang, Malaysia.
[2] MARDI, GPO Box 12301, 50774 Kuala Lumpur, Malaysia.

found to be more appropriate than any other sigmoid function in biological work. Laird *et al.*, (1965) and Laird (1965) demonstrated the usefulness of the Gompertz model in quantitative modeling of the prenatal growth of the guinea pig. Recently, Seber and Wild (1989) had presented that the Gompertz model was the best goodness-of-fit model for animal growth.

Some difficulties are encountered in the estimation of growth curve for sheep. They are the problems related to feeding, physiology, fodder intake under grazing conditions, feed mixing factors and variations in the nutritional requirements. To overcome these problems, body weight of the sheep as a variable was accounted for in an attempt to fit a growth model for sheep. The raw plot of average body weight of sheep versus age for Samarahan and Bau indicated that the body growth followed nonlinear function. In this study, therefore, the Gompertz function was specifically applied for animal growth analysis such as the growth of sheep at Samarahan and Bau experimental sites. And, therefore, the objective of this study was to fit the Gompertz model to sheep growth especially the body weight so that the nonlinear model could be used to predict the apt time for selling and slaughtering of the animals.

MATERIAL AND METHODS

A study on growth as depicted by body weight of Malin crossbred sheep was conducted at Samarahan and Bau experimental sites in Sarawak, Malaysia. The data were collected from experiments arranged in a Randomized Complete Block Design. The variables collected included birth weight, subsequent monthly body weights and age of the sheep. The sample size at Samarahan and Bau were 348 and 73 heads of sheep respectively.

The sheep were grazed on Guinea grass (*Pannicum maximum*) from 08.00 to 16.00 hrs and then they were housed for the night in a wooden shed with raised floor. At night Napier grass (*Pennisetum purpureum*) was provided as fodder for the animals in the shed. Salt lick and fresh water was also made available in the shed. The birth weight and subsequent body weights were taken every 28 days until the animals were 364 days old.

Table 1. Nonlinear least squares iterations for Gompertz model for growth of sheep at Samarahan and Bau experimental sites.

Itera	Location							
	Samarahan				Bau			
	B1	B2	B3	Resid SS	B1	B2	B3	Resid SS
0	10.0000	5.0000	1.0000	585.3323	10.00000	5.00000	1.00000	130.72170
1	16.6049	-1.6551	-0.0147	290.2222	11.20395	0.96433	0.43707	5.29503
2	12.4861	-0.8268	0.2355	82.8917	12.39218	0.48603	0.23196	4.81297
3	13.6054	-0.2003	0.3157	110.3262	14.86583	0.58862	0.18938	1.96257
4	14.6133	0.1570	0.3106	67.2078	15.64333	0.59199	0.18696	0.75035
5	17.4992	0.6463	0.2401	30.9611	15.66845	0.59207	0.18663	0.75015
6	20.7444	0.5814	0.1896	8.1021	15.67038	0.59208	0.18659	0.75015
7	22.6968	0.5880	0.1662	6.1423	15.67058	0.59208	0.18659	0.75015
8	23.3932	0.5938	0.1612	5.8291				
9	23.5030	0.5941	0.1601	5.8265				
10	23.5199	0.5942	0.1560	5.8265				
11	23.5228	0.5942	0.1599	5.8265				
12	23.5233	0.5942	0.1599	5.8265				

Dependent variable : Y
Method : Marquardt
Note : Covergence criteria met

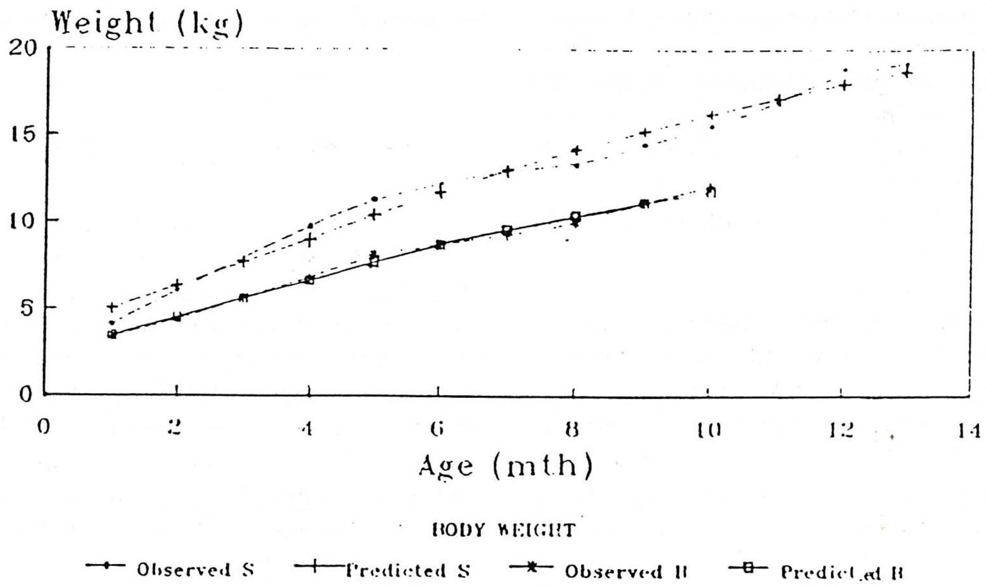

Figure 1. Observed and Predicted Values of Body Weight of Sheep in Sarawak.

The fitting of nonlinear statistical model required initialization values of the parameters so that the nonlinear least squares would arrive at minimum residual sum of squares. Nonlinear least squares is a nonlinear optimization problem for which an optimization algorithm such as Levenburg-Marquardt (Levenburg, 1944; Marquardt, 1963) could be used. The convergent algorithm provided a compromise between the steepest descent and the Gauss-Newton method (Draper and Smith, 1981).

The statistical data analysis procedure used to estimate the parameters of the Gompertz model was SAS nonlinear package, PROC NLIN (SAS, 1985). Gompertz regression modeling was performed on happenstance sheep body weight employing Levenburg-Marquardt compromise convergent algorithm. The initial approximations using ad hoc trial and error procedure for estimating the three parameters of Gompertz curve on both data set were 10.0, 5.0 and 1.0 corresponding to a, B and Y respectively.

The procedure used Levenburg-Marquardt method as a nonlinear iterative algorithm. The algorithm in the estimation issued a message of "convergence criteria met" to indicate global convergence to termination points that have been reached and that the minimization of the residual error sum of squares was attained.

The estimated Gompertz model was also assessed for its goodness-of-fit. Adequacy measures were performed on the nonlinear function of the relationship between sheep body weight and age. The measures constituted t-ratio (Ratkowsky, 1983; Bates and Watts, 1988), confidence intervals of the parameter estimates (Seal, 1964; Younger, 1977; Johnson and Wichern, 1988) asymptotic parameter correlation matrix (Draper and Smith, 1981), analysis of residuals (Ancombe and Tukey, 1963; Weisberg, 1985), and normality test (Seber, 1984).

The data analysis was performed on MARDI's IBM 4381-M11-mainframe computer.

RESULTS AND DISCUSSION

The performance of the Levenburg-Marquardt algorithm for estimating the parameters on data of body weight of sheep from Samarahan and Bau are presented in Table 1. The rate of convergence criterion was met in 12 and 7 iterations. The

Table 2. The relative growth rate (RGR) of sheep at two locations in Sarawak.

Site	Gompertz model	Relative growth rate ($kg\ kg^{-1}\ month^{-1}$)
Samarahan	$Y = 15.6706\ e^{-e^{(0.5921-0.1866X)}}$	0.0962
Bau	$Y = 23.5235\ e^{-e^{(0.5942-0.1599X)}}$	0.0983

X = Age (month)
Y = Body weight

convergent algorithm indicated that the minimum residual sum of squares are in agreement with the regression sum of squares which were equal to 5.8265 and 0.75015, respectively. Global convergence implied that the best estimates of the nonlinear parameters had been procured under the assumption that the fitted model was adequate.

The t-test on the parameters B1 (= a), B2 (=B) and B3 (=Y) depicted that the parameters were well determined. The estimated parameters contributed significantly ($P<0.05$) to the fitted Gompertz model. The asymptotic 95 percent confidence intervals of the parameters were all positive and did not overlap zero. The sign of the confidence intervals, which were all positive, indicated that the estimated parameters were conditioningly well determined. The asymptotic correlation matrix of the parameters indicated that the correlation coefficients were high but not too high (>0.99) to advocate that the fitted model was over parameterized (Draper and Smith, 1981; Bates and Watts, 1988). The assessment, thus, presented the aptness of the Gompertz model for body weight data of sheep.

All nonlinear estimation programs were based on specific assumptions about the disturbance term. Usually the term is additive and normally distributed with zero mean, constant variance and independent between observations (Ratkowsky, 1983; Seber, 1984; and Bates and Watts, 1988).

Plot of residuals versus the fitted model values is a powerful tool in assessing the goodness-of-fit of the fitted model (Anscombe and Tukey, 1963). Consequently, analysis of residual was performed on the body weight of sheep fitted model. The scattered points of the residual variance on predicted response (body weight) were constant and independent. The residual analysis depicted that the expectation function was adequately fit. The plotting of observed and predicted body weight of the sheep using the above model were close to each other (Figure 1). It was evident that the fitted model had described adequately the sheep body weight data. The relative growth rate of sheep body weight at Samarahan and Bau experimental sites were determined to be 0.0983 and 0.0962 $kg\ kg^{-1}\ month^{-1}$ (Table 2).

CONCLUSION

It is evident that the Gompertz curve is aptly fit for sheep body weight as measure of growth. However, it is suggested that other growth models may also be examined. The relative growth rate of the sheep provided very pertinent information on the most suitable time for selling the animals and/or possible age to slaughter the animals for food.

REFERENCES

Anscombe, F.J. and Tukey, J.W. 1963. The examination of residuals. *Technometrics*, 5:141-160.

Bates, D.M. and Watts, D.G.1988. *Nonlinear Regression Analysis and its Application*, New York: Wiley.

Bliss, C.I. 1970. *Statistics in Biology*. Vol. 2, New York: McGraw-Hill.

Couston, D.R. and Venus, J.C. 1981. *The Biometry of Plant Growth*, London: Edward Arnold.

Draper, N.R. and Smith, H. 1981. *Applied Regression Analysis*. 2nd Ed., New York: John Wiley.

Gompertz, B. 1825. *Philos. Trans. R. Soc. A.*, 115:513-580.

Johnson, R.A. and Witchern, D.W. 1988. *Applied Multivariate Analysis*, 2nd Ed, Englewood Cliffs, New Jersey: Prentice Hall.

Laird, A.K. 1965. Dynamics of Relative Growth. *Growth*, 29:249-263.

Laird, A.K. Tyler, S.A. and Barton, A.D. 1965. Dynamics of normal growth. *Growth*, 29:233-248.

Levenburg, K. 1944. A method for the solution of certain nonlinear problems in least squares. *Quart. Appl. Math.*, 2:164-168.

Marquardt, D.W. 1863. An algorithm for least squares estimation of nonlinear parameters. *J. Soc. Indust. Appl. Math.*, 11:431-439.

Medawar, P.B. 1940. Growth, growth energy and aging of chicken's heart. *Proc. Roy. Soc. London*, 129:332-355.

Ratkowsky, D.A. 1983. *Nonlinear Regression Modelling*. New York: Marcel Dekker.

SAS. 1985. *SAS User's Guide: Statistics*. Version 5 Edition. Cary, North Carolina: SAS Institute.

Seal, H.L. 1964. *Multivariate Statistical Analysis for Biologists*. London: Metheum.

Seber, G.A.F. 1984. *Multivariate Observations*. New York: John Wiley.

Seber, G.A.F. and Wild, C.J. 1989. *Nonlinear Regression*. New York: John Wiley.

Weisberg, S. 1985. *Applied Linear Regression*. 2nd Ed. New York: John Wiley.

Younger, M.S. 1979. *Handbook for Linear Regression*. North Scituate, Massachusetts: Duxbury.