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Best Fitting Methods for The Mud Profile Equations

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ABSTRACT In order to understand the dynamics of shoreline changes due to natural and anthropogenic causes, it is imperative for a coastal manager to comprehend the shore profile characteristics which are dependent on the sediment-wave interaction and can be depicted in a profile equation. Moreover, it is possible to derive the power form for the profile equation of a sandy coast based on the argument of wave energy dissipation per unit bed area and unit time. By using this same argument and considering the phenomenon that the main cause of wave damping over a muddy coast is due to energy absorption by the soft mud bottom, the mud profile equation can also be formulated. The aim of this study was to observe the mud profile equation geometry using best fitting method and to compare the characteristic features of the mud profiles using the field observation data. Shore profile data were measured from the muddy coast of Pantai Cermin in the eastern coast of North Sumatera Province. The data obtained were fitted to both the sand and mud profile equations. The procedures and results of the two best fitting methods, the nonlinear regression and the least square based trial and error search, were exhibited and compared. Several noteworthy features of the mud profile equation were found to be the same with the sand profile equation in describing the profile data. In order to provide a better profile and shoreline stabilization, it is recommended to use more complete observation data and good knowledge of shore profile by the coastal manager.

KEYWORDS Best Fitting; Mud Profile; Shore Profile; Sand Profile; Trial-error Method

1 INTRODUCTION

A shore profile is produced from nearshore forces acting on the bed sediments across the active sediment mobility zone with the coast generally consisting of mud or sand. In an episode of stormy weather conditions, the bed sediments interact with waves which is the typical predominant forces in an open coast to dissipate wave energy and produce a (dynamic) equilibrium profile (Dean & Dalrymple, 2002).

The main wave energy dissipation over a sandy coast has been commonly recognized to be due to the turbulent breaking mechanism across the surf zone. In contrast, over a muddy coast, the wave energy is damped due to the existence of a soft mud bottom (Tarigan, 1996; Tarigan, 2002). The resulting mud profiles are typically milder and most plausibly longer than the sandy profiles. Therefore, their proposed profile equations are expected to be different as a reflection of their distinctive characters.

The objective of this paper was to show the geometry of the mud profile equations through the best fitting method and to also compare the characteristic features of the mud profiles with those of the sand using field data and observation.

2 BACKGROUND THEORY

A shore profile is a geometry of the variation in the water depth of the offshore perpendicular to the shoreline. In an equilibrium condition, it is theoretically the product of the balance between the destructive and constructive forces acting across the profile. Due to the natural continuous change in the incident wavefield and water level, the equilibrium profile is considered to be the dynamic concept (Dean & Dalrymple, 2002).

Most of the shore profile equations are derived based on the argument of wave energy conservation. This means a profile responds dynamically towards the equilibrium shore profile if its sediment size is considered to have the ability to withstand a given level of energy dissipation per unit water volume or area.

2.1 Sand Profile Equations

Bruun (1954) and Dean (1977) were among the first to propose a power form for a profile with coarse-grained sediments using Equation (1).

$$h = Ay^{n} \tag{1}$$

where h is water depth (m), A *is* profile scale parameter, y *is* distance from shoreline (m) and n *is* constant.

Dean (1977) found it is possible to derive the value of n = 2/3 from the argument of equilibrium beach profile produced by the wave energy dissipation per unit volume D_{eq} as described in Equation (2).

$$D_{eq} = \frac{dP}{hdy} \tag{2}$$

The energy flux P can also be expressed using the shallow water assumption of the linear wave theory as shown in Equation (3).

$$P = \frac{1}{8}\rho g H^{2}\sqrt{gh} \tag{3}$$

where ρ is the density of the fluid. It should be noted that the main wave energy dissipation is due to the turbulence generating the break over the surf zone with the wave height expressed in terms of the breaking index, k, as shown in Equation (4).

$$H = kh \tag{4}$$

The dissipation D_{eq} then becomes Equation (5).

$$D_{eq} = \left[\frac{5}{16}\rho g^{2/3}k^{2}\right]h^{1/2}\frac{dh}{dy} \tag{5}$$

which can be solved to have the sand profile equation as Equation (6).

$$h = Ay^{2/3} \tag{6}$$

The value of A is given in Equation (7).

$$A = \left(\frac{24D_{eq}}{5\rho g^{3/2}k^{2}}\right)^{3/2} \tag{7}$$

2.2 Mud Profile Equations

Mud profiles are generally located on low laying coastal areas, especially in the estuary vicinities where abundant fine sediments are suspended and dispersed toward the coastal waters. The muddy coasts with soft sediment bottom are typically broad, flat, and shallow forming mild profiles (Tarigan & Nurzanah, 2016).

Lee (1995) and Lee & Mehta (1997) argued the breaking wave is not the main energy dissipation across the mud profile and also reported the thickness of the fluid mud affects the absorption of wave energy. Therefore, the wave damping is expressed in Equation (8).

$$H_{(y)} = H_{0}e^{-Ki(y_0 - y)} \tag{8}$$

where H_0 is the height of the incident wave at $y = y_0$, y_0 is the length of the active profile, and K_i is the wave damping coefficient. This equation is substituted into a uniform wave energy dissipation equation per unit area E_{eq}, as shown in Equation (9).

$$E_{eq} = \frac{d}{dy}\left(EC_{g}\right) \tag{9}$$

With shallow water condition $C_g = \sqrt{gh}$, Equation (8) combined with Equation (9) produce Equation (10).

$$\frac{\rho g^{3/2}H_{0}^{2}}{2}\frac{d}{dy}\left[e^{2K_i(y-y_0)}\sqrt{h}\right] = E_{eq} \tag{10}$$

This equation is integrated from the coastline (0,0) to the point (*y, h*) as Equation (11) (Mehta, 2014).

$$\int_{0,0}^{y,h} d\left[e^{2K_l(y-y_0)}\sqrt{h}\right] = \int_0^y \frac{2E_{eq}}{\rho g^{3/2}H_0^2}dy \quad (11)$$

which can be solved as Equation (12).

$$e^{2\overline{k_l}(y-y_0)}\sqrt{h} = \frac{2E_{eq}}{\rho g^{3/2}H_0^2}y \quad (12)$$

where $\overline{k_l}$ represents the average wave damping coefficient. This expression should satisfy the boundary condition, i.e., $h = h_0$ at $y = y_0$. So that

$$H_0^2 = \frac{2_{Eq}y_0}{\rho g^{3/2}\sqrt{h_0}} \quad (13)$$

Substituting Equation (13) to Equation (12) yields the initial mud profile equation

$$h = h_0 e^{4\overline{k_l}(y_0-y)}\left(\frac{y}{y_0}\right)^2 \quad (14)$$

Lee (1995) found Equation (14) did not match effectively with the field data on the nearshore of the profile and this was suggested to be due to the mechanism of wave dissipation apart from those absorbed through wave energy by mud, for example, the turbulence due to breaking waves. Lee (1995), therefore, added the correction term C_N to Equation (14) to obtain *h* required to solve this problem.

$$C_N = Fye^{-\beta y}$$

$$h = Fye^{-\beta y} + h_0 e^{4\overline{k_l}(y_0-y)}\left(\frac{y}{y_0}\right)^2 \quad (15)$$

where in Equation (15), *F* is the bottom slope at the shoreline and β is the offshore extent of the combined influence of the slope at the shoreline and scour due to wave breaking. Finally, to maintain consistency at the boundary conditions of $h = h_0$ at $y = y_0$, Lee (1995) obtained the final shape for the mud profile geometry, as shown Equation (16).

$$h = Fye^{-\beta y} + \left(h_0 - Fye^{-\beta y}\right)e^{4\overline{k_l}(y_0-y)}\left(\frac{y}{y_0}\right)^2 \quad (16)$$

This geometry was concluded to have retained the analytic properties of the model stated by Equation (14). This study suggested a different correction term C_N to improve the performance of the equation near the shoreline as shown in Equation (17).

$$C_N = F(1-e^{-\beta y}) \quad (17)$$

Therefore, Equation (14) becomes equation (18)

$$h = F(1-e^{-\beta y}) + h_0 e^{4\overline{k_l}(y_0-y)}\left(\frac{y}{y_0}\right)^2 \quad (18)$$

In order to maintain consistency at the boundary conditions, $h = h_0$ at $y = y_0$, the modified mud profile equation obtained shown in Equation (19).

$$h = F(1-e^{-\beta y}) + (h_0 - Fye^{-\beta y})e^{4\overline{k_l}(y_0-y)}\left(\frac{y}{y_0}\right)^2 \quad (19)$$

2.3 Nonlinear Regression and Trial Error Method

The nonlinear regression was based on the Gauss-Newton method and the Taylor series. The Gauss-Newton method is an algorithm to minimize the sum of squares of the difference between the data and the nonlinear equation. Moreover, the key concept underlying this technique is the use of Taylor series expansion to express the original nonlinear equation in a linear, approximate form after which the least-squares theory was used to obtain new estimates of parameters aimed at minimizing residuals (Chapra & Canale, 2015).

To illustrate how this process, the relationship between nonlinear equations and data was first generally stated as expressed in Equation (20).

$$y_i = f(x_i; a_0, a_1, \ldots, a_m) + e_i \quad (20)$$

where y_i = the measured value of the dependent variable, $f(x_i;a_0,a_1,...,a_m)$ = the equation serving as a function of the independent variable x_i and the nonlinear function of the parameter $a_0,a_1,...,a_m$ and e_i = a random error. For convenience, this model can be expressed in a simple form by eliminating the parameters to become Equation (21).

$$y_i = f(x_i) + e_i \tag{21}$$

It is also possible to extend the nonlinear model in the Taylor series around the values with a limit up to the first derivative. For example, in the case of two parameters.

$$f(x_i)_{j+1} = f(x_i)_j + \frac{\partial f(x_i)_j}{\partial a_0}\Delta a_0 + \frac{\partial f(x_i)_j}{\partial a_1}\Delta a_1 \tag{22}$$

where j is initial value, j + 1 is the prediction, $\Delta a_0 = a_{0,j+1} - a_{0,j}$ and $\Delta a_1 = a_{1,j+1} - a_{1,j}$. Then by substituting Equation (22) to Equation (23) the following is obtained:

$$y_i - f(x_i)_j = \frac{\partial f(x_i)_j}{\partial a_0}\Delta a_0 + \frac{\partial f(x_i)_j}{\partial a_1}\Delta a_1 + ei \tag{23}$$

or in the matrix form in Equation (24).

$$\{D\} = [Z_j]\{\Delta\hat{A}\} + \{E\} \tag{24}$$

where *[Zj]* is the partial derivative matrix of the function evaluated at the initial value,

$$[Z_j] = \begin{bmatrix} \partial f_1/\partial a_0 & \partial f_1/\partial a_1 \\ \partial f_2/\partial a_0 & \partial f_2/\partial a_1 \\ . & . \\ \partial f_m/\partial a_0 & \partial f_m/\partial a_1 \end{bmatrix}$$, *m* is number of data,

and $\partial f_i / \partial a_k$ is the partial derivative of the function with respect to the *k-th* parameter evaluated at the *i-th* data point. Vector *{D}* contains the difference between measurement and function value, $\{D\} = \begin{Bmatrix} y_1 - f_{(x_1)} \\ y_2 - f_{(x_2)} \\ . \\ : \\ y_m - f_{(x_m)} \end{Bmatrix}$ and vector $\{\Delta\hat{A}\}$ contains the difference of parameter value,

$$\{\Delta\hat{A}\} = \begin{matrix} \Delta a_0 \\ \Delta a_1 \\ . \\ : \\ \Delta a_m \end{matrix}$$

Applying the theory of least squares to Equation. (24) produces the following normal equation.

$$[[Zj]^T[Zj]]\{\Delta\hat{A}\} = \{[Z_j]^T\{D\}\} \tag{25}$$

Therefore, the approach included the completion of Equation (25) for $\{\Delta\hat{A}\}$, to calculate the value of parameters, $\Delta a_{0,j+1} = a_{0,j} - \Delta a_0$ and $\Delta a_{1,j+1} = a_{1,j} - \Delta a_1$. The procedure was repeated until the solution converged such that the standard error reached the value below an acceptable stopping criterion.

$$S = \sqrt{\frac{\sum(y_i - f_{(x_i)})^2}{m-1}} \tag{26}$$

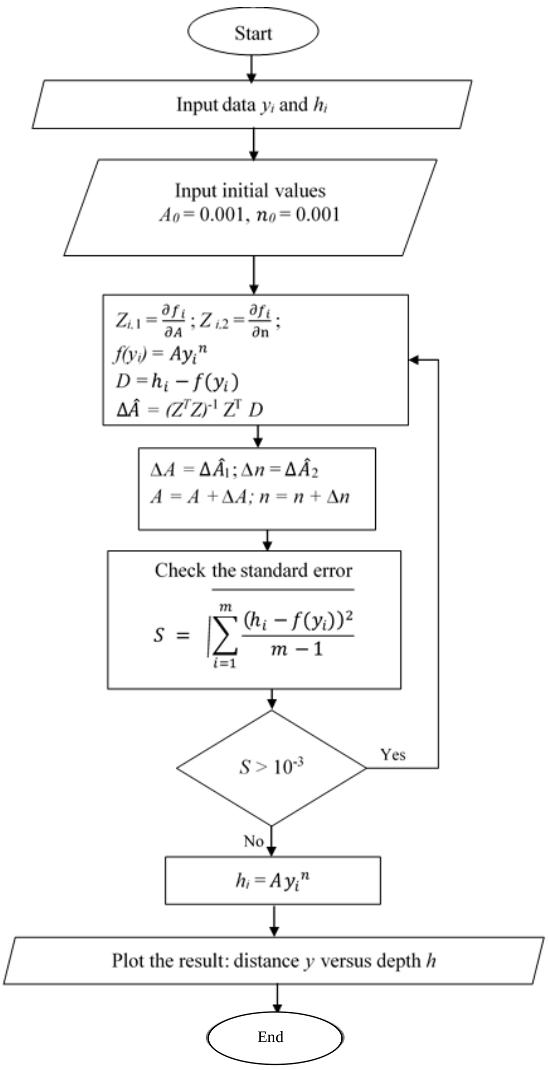
This trial and error method was conducted by systematically changing or increasing the values of the parameters with a very small interval, for example 0.001, in the valid ranges considered. At each step of computation, the standard error of Equation (26) was calculated and the step with the smallest value of *e* was identified and defined as the best solution containing the best parameters. Figure 1 shows the flowchart of the best fitting of the profile equation based on the field data using the two methods previously described.

3 METHODS

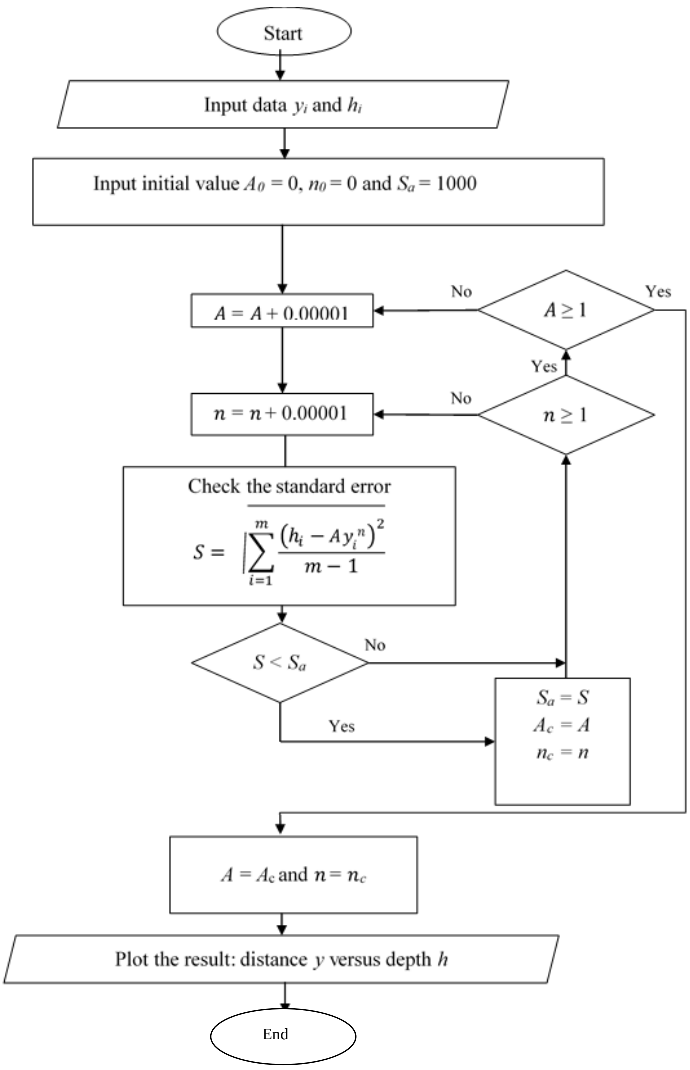
3.1 Field Site and Measurement

The field site was the muddy coast of Pantai Cermin, Kota Pari village, Pantai Cermin district, Serdang Bedagai county. The locals name the site Pantai Mutiara and was found to be located on the eastern coast of North Sumatera Province facing the Strait of Malacca. It is about 43 km from Medan, the capital city of Sumatera Utara Province. Its geographic location is in the vicinity of 3° 39' 46" northern latitude and 98° 57' 54" eastern longitude as shown in Figure 2.

The profile was measured using a geodetic GPS with the RTK (real-time kinematic) method at an accuracy up to 5 mm in horizontal and vertical positions. Moreover, the coordinates *(x, y, z)* were provided in the UTM projection system on the 47 N zone.Table 1 shows the field data in terms of distance from shoreline and elevation. It is important to note that the minus sign indicates the point of measurement is already in the water with reference to the local datum which is approximately the mean high-water level.



(a)



(b)

Figure 1. Flowchart of best fitting of the profile equation using (a) nonlinear regression and (b) trial and error methods

Figure 2. Location of the field site

Table 1. Field data

No.	Distance from shoreline y (m)	Elevation h (m)
1	-46.443	0.487
2	-30.895	0.596
3	-12.432	1.388
4	-8.498	1.379
5	-4.438	0.638
6	0.000	0.000
7	1.294	-0.186
8	5.262	-0.617
9	8.663	-1.013
10	16.998	-1.043
11	28.490	-0.981
12	57.947	-1.027
13	86.417	-1.002
14	162.321	-1.165
15	223.429	-1.231
16	277.884	-1.342
17	327.736	-1.494
18	359.460	-1.591
19	401.065	-1.701

4 RESULTS AND DISCUSSION

Two methods were used to fit the profile equations for sand and mud with the field data and they include the nonlinear regression and trial and error methods as shown in Figures 3 and 4. The results obtained were, therefore, compared to determine the best fitting.

The sand profile equation was fitted to two different sets of data including the beach face and the whole part. Figure 3a shows the result of best fitting for the beach face data using the nonlinear regression while Figure 3b shows for the trial and error method. Both methods were observed to have yielded approximately the same order of accuracy. A noteworthy feature in the two figures is the fact that the sand profile equation has the ability to represent only near the shoreline part of the profile data. This is in agreement with the field observation which showed sand with a median size of 0.5 – 1.00 mm have been typically deposited on the beach face of the profile.

Figures 3c and 3d also indicate the result of best fitting for the sand profile equation involving the whole data using nonlinear regression and trial and error methods, respectively. As noted in the previous case, both methods also produced almost the same order of accuracy. A noteworthy feature in these two figures is the fact that the value of n is very low compared with the suggested value n = 2/3. This means the profile is very mild, especially on the main portion below the beach face where mud is typically deposited.

Figures 4a and 4b show the result of best fitting for the mud profile equation of Equation (16) with the whole data using nonlinear regression and trial and error methods, respectively. The nonlinear regression was discovered to have yielded a slightly better accuracy but both exhibited significant discrepancy near the shoreline. This indicates the equation has a drawback in characterizing the beach face. This was, therefore, improved in Figures 4c and 4d in which the results of best fitting the mud profile equation of Equation (19) with the whole data using nonlinear regression and trial and error methods were provided respectively. As indicated by the lowest standard errors, this equation had the best performance but there is need to be aware of the ranges of valid values for the parameters F, β and $\overline{k_i}$ to achieve the best fitting.

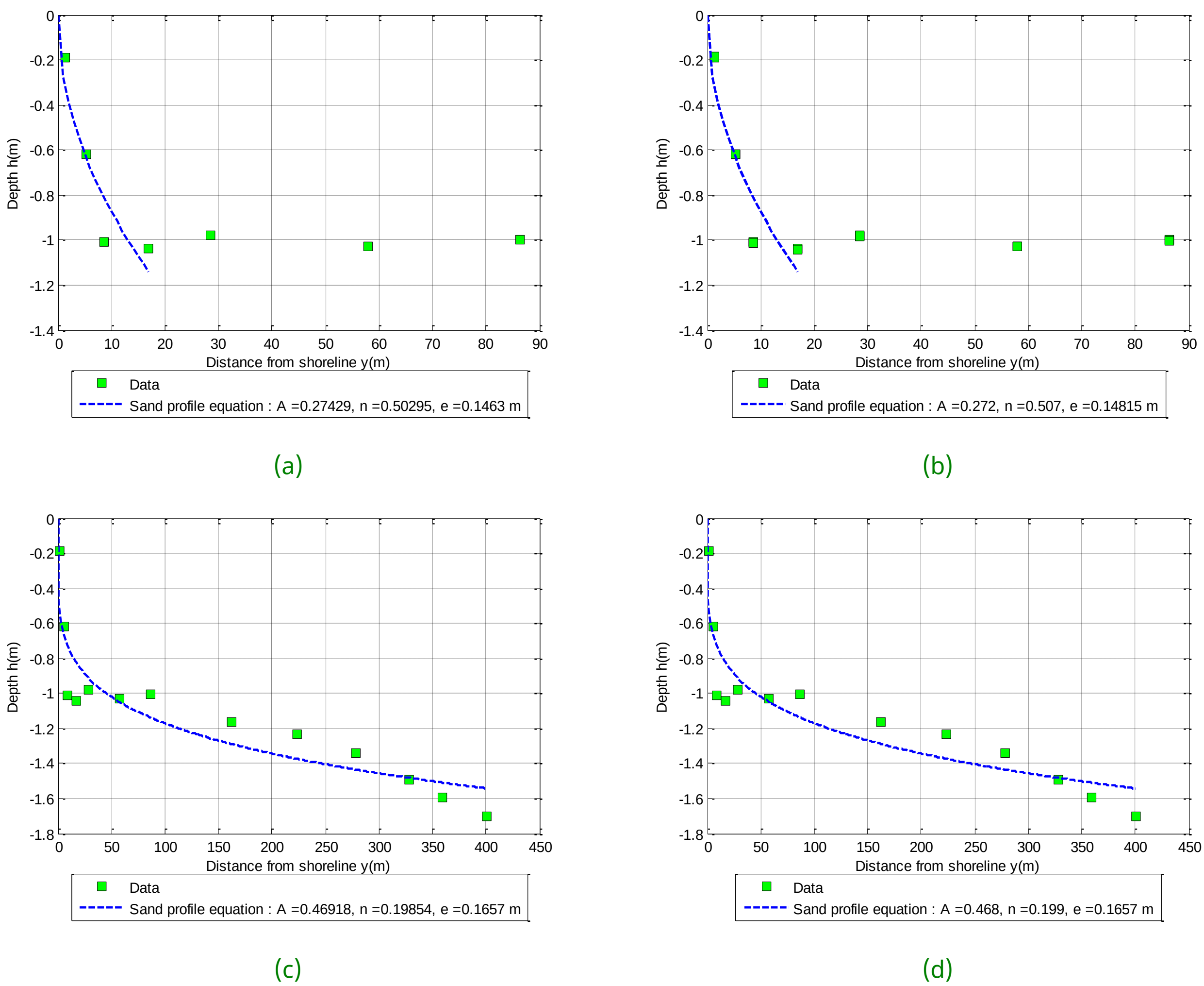
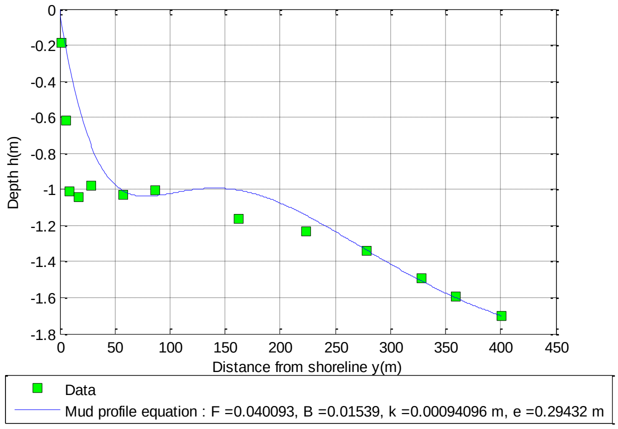
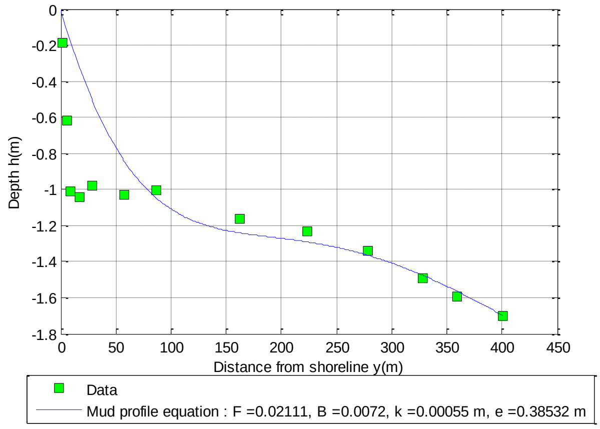


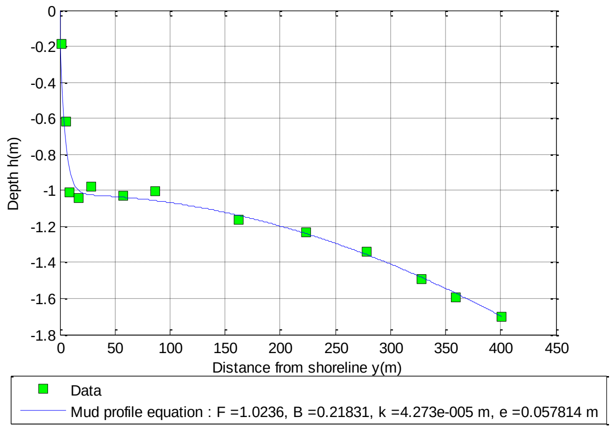
Figure 3. Results of best fitting of sand profile equation at the beach face using (a) nonlinear regression and (b) trial and error methods and with all of the field data using (c) nonlinear regression and (d) trial and error methods



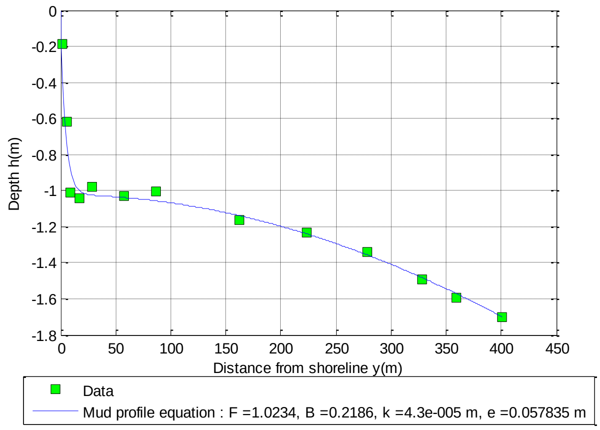
(a)



(b)



(c)



(d)

Figure 4. Results of best fitting of the mud profile equation of Equation (16) using (a) nonlinear regression and (b) trial and error methods and best fitting of Equation (19) using (c) nonlinear regression and (d)trial and error methods

5 CONCLUSION

The following points of conclusion were drawn from the results:

1) Both methods of nonlinear regression and trial and error methods had almost the same accuracy and in order to achieve the best fit, there is need to be aware of the ranges of the valid values for the parameters involved.
2) The sand profile equation was good only for the beach face part where sand is typically deposited.
3) The mud profile was very mild to the extent the n value of the sand profile equation, suggested commonly to be n = 2/3, was meaningless.
4) The mud profile equation of Equation (16) suggested by Lee (1995) was derived based on the assumption of wave damping due to the existence of soft, mud bottom, a characteristic feature different from the sand profile equation. However, it performed troublesomely on the foreshore part of the profile.
5) The modified mud profile equation of Equation (19) improved the drawback of Equation (16) and yielded the best fitting with the whole data, including the steep foreshore of the profile.

It is, therefore, recommended that more profile data be obtained to examine the applicability of the mud profile equations. The knowledge of shore profiles is also essential to a coastal manager dealing with profile and shoreline stabilization.

DISCLAIMER

The authors declare no conflict of interest.

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