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Comparison of Two Corrector Surface Models of Orthometric Heights from GPS/Levelling Observations and Global Gravity Model

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ABSTRACT

The advent of space-based measurement systems such as the Global Positioning System (GPS) offers a new alternative in orthometric height determination over conventional spirit levelling. The ellipsoidal height (h) obtained from GPS observations can be transformed into orthometric height if the geoidal height (N) is known from a national gravimetric geoid model. However, the lack of a national geoid model in Nigeria hinders the use of the method. This study compares two corrector surface models of orthometric heights from GPS/levelling observations and the Global Gravity Model. Model A (7-parameter) and Model B (8-parameter) are based on the general 7-parameter similarity datum shift transformation. A network of twenty-one (21) GPS/levelling benchmarks within the study area were used and their geoidal heights were computed using GeoidEval utility software with reference to Global Gravitational Model (EGM08). Least squares adjustment was used to compute the coefficients of the models. Root mean square error (RMSE) was used to assess the accuracy of the models with model A having RMSE=0.171m and model B having RMSE=0.169m. Model B with the lowest RMSE is hence the better of the two models. The t-test and hypothesis tests conducted at a 95% confidence level, however, revealed that the two models did not differ significantly. The study shows that the use of a corrective surface to combine the gravity field model EGM08 with GPS/levelling significantly improves the determination of heights as observed from GPS in the study area.

Keywords: Accuracy, Ellipsoidal heights, Geoid model, GPS, Orthometric heights, Hypothesis test

1. Introduction

Accurate determination of the height of features on, above, or below the earth surface is germane in a geodetic framework be it ellipsoidal height, orthometric height, or other systems of height in geodesy. Among these heights, orthometric height which is considered natural is required for surveying and mapping projects, engineering survey projects, geophysical projects, and other geospatial applications because of its relationship with the ocean (water body) and earth's gravity field (Aleem et al., 2011). The traditional spirit levelling whose operation is tedious, time-consuming, expensive, and requires more manpower has been used over a century as the conventional method for the computation of orthometric heights due to its simplicity, effective operation, and remarkable precision. The operation also requires points whose heights need to be determined be interconnected by a series of levelling lines which makes the operation prone to many systematic errors which are difficult to discover and remove. However, several efforts have been made in recent times to develop alternative methods and technologies to be incorporated into levelling to suit current needs (Aleem et al., 2011).

The advent of the Global Navigation Satellite System (GNSS) particularly the Global Positioning System (GPS) has enhanced accurate determination of points position. The position of points derived from GPS observations are usually computed in a three-dimensional Cartesian coordinate system and are then transformed into the more recognizable geodetic latitudes, longitudes, and ellipsoidal heights (Abdullah, 2010). The GPS constitutes the best-known satellite navigation system that offers independent geospatial positioning with global coverage. Therefore, the many benefits provided by the GPS have made it a suitable alternative over traditional spirit levelling. GPS offers a new alternative for the accurate determination of orthometric height over a comparatively short period. The ellipsoidal height derived from GPS observations can be converted into orthometric height if the geoidal height is known (Abdullah, 2010). Many countries of the world have adopted this approach of deriving orthometric height because it is fast, less tedious, and relatively cheap in contrast with the geodetic levelling approach (Abdullah, 2010).

The optimal combination of GPS-derived ellipsoidal heights with gravimetrically derived geoidal heights for the determination of orthometric heights with reference to a vertical datum is known as GPS-levelling. The method is given by (Eteje et al., 2018; Oluyori et al., 2018; Ono, 2009; Uzun and Cakir, 2006) as shown in the relationship below.

$$H = h - N \tag{1}$$

A major problem of using GPS-levelling as a means of establishing heights with respect to a local vertical datum is that it is dependent on the level of accuracy of the ellipsoidal and geoidal height data. However, the relationship given by eq. (1) is hindered by numerous errors, systematic distortions, and datum inconsistencies inherent among the three heights data (Fotopoulous, 2003; and Fotopoulous et al., 2001a). Therefore, to solve the problem of datum inconsistencies and any systematic distortions that are in the height data sets, a parametric corrector surface model can be incorporated in eq. (1) for the integration of these different height systems (Fotopoulous, 2003; and Shrestha et al., 1993). This study, therefore, focuses on the comparison of two corrector surface models for further transformation of the GPS-derived ellipsoidal heights into orthometric heights within appreciable accuracies. Because appropriate gravimetric data are not available in Nigeria, Earth Gravity Model 2008 (EGM08) which is a satellite gravimetric-based solution is fitted into GPS/levelling in the study area.

1.1. The objectives of the study

The focus of this study is to compare two corrector surface models for orthometric height in Akure, with a view to recommending which model to adopt by the GPS user community for different applications within appreciable accuracies. The objective includes determination of ellipsoidal height (h) of control points using DGPS observations; computation of geoidal height (N) and developing Microsoft excel program for interpolation of N; determination of the orthometric height and comparison of the results obtained from the two models by using t-test statistics.

1.2. Overview of corrector surface models under investigation

The fundamental relationship between the ellipsoidal height derived from GPS observations and orthometric height with respect to a vertical datum established from traditional spirit levelling and gravity survey is given by Moka and Agajelu (2006) as,

$$h - H - N = 0 \tag{2}$$

Where:

h = Ellipsoidal height,
H = Orthometric height
N = Geoidal undulation or Geoidal height

The geometrical relationship between the three height types is based on the basis that given any two of the heights, the third can be determined through simple manipulation of eq. (2). In practice, the implementation of the eq. (2) is more complicated due to numerous factors that cause discrepancies when combining the different height data sets (Aleem et al., 2011, Fotopoulous, 2003; Kearsley et al., 1993; Rummel and Teunissen, 1989). The key part of these discrepancies is usually attributed to the datum inconsistencies inherent among the height types which are usually referred to a slightly different reference surface and systematic effects primarily caused by long-wavelength geoid errors, poorly modelled GPS errors, and over-constrained levelling network adjustments. These effects can be dealt with, by using a corrector surface model in the combined adjustment of the heights (Fotopoulous, 2003).

The basic model for the corrector surface is given by Fotopoulous (2003) as,

$$q_i=h_i-H_i-N_i=a_i^T x+v_i \tag{3}$$
$$a_i^T x=N^{GPS}-N^{Grav} \tag{4}$$

Where h_i, H_i, N_i are previously defined, x is a n ×1 vector of unknown parameters (where n is the number of the GPS/levelling points), a is a n × 1 vector of known coefficients, v_i represents a residual random noise term, N^{GPS} is the geoid heights derived via GPS and N^{Grav} denotes the geoid heights from a regional gravimetric geoid model or a global gravitational model.

The corrector surface model $a_i^T x$ is supposed to describe the datum inconsistencies and systematic errors inherent in the heterogeneous height data sets. This study focuses on two corrector surface models based on the general 7-parameter similarity datum shift transformation and they are.

1. 7 Parameter Model (Model A)

The seven-parameter model based on differential similarity transformation is defined by (Isioye et al., 2011; Raizner, 2008; and Fotopoulous, 2003) as shown in the relationship below.

$$a_i^T x = x_1 cos\varphi_i cos\lambda_i + x_2 cos\varphi_i sin\lambda_i + x_3 sin\varphi_i + x_4\left[\frac{sin\varphi_i cos\varphi_i sin\lambda_i}{W_i}\right] + x_5\left[\frac{sin\varphi_i cos\varphi_i cos\lambda_i}{W_i}\right] + x_6\left[\frac{1-f^2 sin^2\varphi_i}{W_i}\right] + x_7\left[\frac{sin^2\varphi_i}{W_i}\right] \tag{5}$$

Where φ_i, λ_i are the latitude and longitude of the GPS-levelling points respectively.

2. 8-Parameter Model (Model B)

The more complicated non-rigid similarity transformation model with an eight-parameter structure is given by (Isioye et al., 2011; Raizner, 2008; and Fotopoulous, 2003) as eq. (6).

$$a_i^T x = x_1 cos\varphi_i cos\lambda_i + x_2 cos\varphi_i sin\lambda_i + x_3 sin\varphi_i + x_4\left[\frac{sin\varphi_i cos\varphi_i sin\lambda_i}{W_i}\right] + x_5\left[\frac{sin\varphi_i cos\varphi_i cos\lambda_i}{W_i}\right] + x_6(aW_i + h_i) + x_7\left[\frac{1-f^2 sin^2\varphi_i}{W_i}\right] + x_8\left[\frac{sin^2\varphi_i}{W_i}\right] \quad (6)$$

Where the quantity W_i is given by the relationship.

$$W_i = \sqrt{1 - e^2 sin^2\varphi_i} \quad (7)$$

And the quantities a, f, e^2 in the above formulae correspond to the semi-major axis, flattening and eccentricity, respectively, of the reference ellipsoid.

2. Data and Methods

2.1. Study area

The study area is Akure, the capital of Ondo State in the South-Western part of Nigeria. It lies between latitude 7o 15'N to 7o 30'N and longitude 5o 15'E to 5o 25'E. The twenty-one (21) network of previously established benchmarks that were selected is located in Akure South Local Government Area of Ondo State.

2.2. Data

A network of twenty-one (21) GPS/levelling benchmarks which has been previously established within the study area were re-coordinated using South DGPS. The DGPS was used to acquire positional data and ellipsoidal heights of the selected benchmarks in static mode (2 hours) per station with five seconds epoch rate.

2.3. Geoidal heights

The geoidal height (or geoidal undulation) can be referred to as the separation of the reference ellipsoid with the geoid surface measured along the ellipsoidal normal. It is also the distance between the ellipsoid and the geoid. To obtain this distance between the ellipsoid and the geoid for the twenty-one (21) selected stations, Earth Gravity Model 2008 (EGM2008 or EGM08) is used to interpolate geoidal height.

The Earth Gravity Model 2008 (EGM08) is a global geopotential model publicly released towards the end of 2008 by the United State National Geospatial-Intelligence Agency (NGA), EGM Development Team. This gravitational model is complete to spherical harmonic degree and order 2159 and contains additional coefficients extending to degree 2190 and order 2159 (Pavlis et al., 2008).

This model represents a very significant improvement in precision over earlier geoid models. It is available in 1, 2.5 and 10 minute grid sizes. To determine the geoidal height of the selected benchmarks, GeoidEval utility software was implemented. This software computes the geoidal height with reference to WGS84 ellipsoid using interpolation in a grid of values for the earth gravity models EGM84, EGM96 or EGM2008. The statistics for the absolute differences between the geoidal height obtained from EGM08 and GPS-derived geoidal height is given in Table (1).

Table 1 The statistics of differences between geoidal height from EGM08 and GPS for the selected stations

	N^{GPS}	N^{EGM08}	Differences
Max. value (m)	14.820	24.805	-9.848
Min. value (m)	13.394	24.612	-11.331
Mean (m)	13.879	24.737	-10.857
Std. deviation (m)	0.409	0.069	0.448
No of stations =21			

2.4. Mathematical model formulation

In this study, the method of observation also called the method of parameters or method of variation of parameters is used to formulate the model. Thus, an observation equation is written for each observation in the form given below.

$$V = AX - L^b \quad (8)$$

Where A is the design matrix and V is the vector of residuals

X is the vector of unknown parameters

L^b is the vector of observations (i.e., $\Delta N = N^{GPS} - N^{EGM08}$)

2.5. Solving the models using least squares method

The method of least squares is a standard approach in an over-determined system to the approximate solution. In this study, the method of Least squares has been employed to determine the parameters in Model A (7-Parameter Model) and Model B (8-Parameter Model). The least squares method is based on the minimization of the sum of square of residuals to a minimum. The solution of the formulation of the least squares is given below.

$$X = (A^T PA)^{-1}(A^T PL) \quad (9)$$

$$X = (A^T A)^{-1}(A^T L) \quad (9a)$$

Equation (9a) is for unit weight based on equal reliability of observations.

Standard error of observations (σ) is expressed as:

$$\delta = \sqrt{\frac{v^2}{n-1}} \quad (10)$$

The parameters of the two corrector surface models were determined with least squares method using MatLab software. The values of the estimated transformation parameters for Model A and Model B are given in Table 2 and 3 below.

Table 2 Values of estimated transformation parameters (Model A)

Parameter	Parameter value (m)
X1	-14914484.89141
X2	1460781.83065
X3	-184717952.51612
X4	-5239051.140414
X5	187835693.63591
X6	14509177.58980
X7	2768812.95557
σ=0.1568m	

Table 3 Values of estimated transformation parameters (Model B)

Parameter	Parameter value (m)
X1	-15379433.09645
X2	1157064.28714
X3	-176195933.66856
X4	-3968806.05981
X5	178999796.36889
X6	-0.00304
X7	15033769.24294
X8	2343063.27969
σ=0.1563m	

2.6. Estimation of orthometric heights

A program was written for the corrector surface models $(a_i^T x)$ using Microsoft Excel 2016 and the orthometric height (H) for the twenty-one (21) selected control points used in the study was estimated as follows:

$$H_i = h_i - N_i^{EGM08} - a_i^T x \quad (11)$$

The results of the estimated orthometric heights from the selected benchmarks are presented in Table (4) and the statistics of the differences between known orthometric heights of the points and their estimated values for the two corrector surface models are presented in Table (5) in which the Root Mean Square Error (RMSE) is estimated as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_{obs,i} - X_{model,i})^2}{n}} \quad (12)$$

Where X_{obs} is the known values, X_{model} is the modelled values, and n is the number of points.

Table 4 Estimated orthometric height for the selected control points

Control Points	Known Orthometric Height (H)m	Estimated Orthometric Heights (H)m	
		Model A	Model B
GPSA72S	346.470	346.449	346.459
GPSA73S	345.146	345.019	345.032
GPSA75S	338.388	338.472	338.467
GPSA76S	336.666	336.715	336.706
GPSA77S	334.651	334.703	334.687
GPSA78S	337.365	337.340	337.332
GPSA79S	342.538	342.630	342.637
GPSA80S	345.831	345.829	345.843
FG28	345.817	345.816	345.825
FG29	340.115	340.080	340.069
GPSA81S	338.215	338.351	338.346
GPSA82S	334.102	333.958	333.947
GPSA83S	349.765	349.620	349.653
GPSA84S	345.840	345.798	345.804
GPSA85S	339.197	339.243	339.220
GPSA45S	332.621	332.860	332.859
GPSA46S	331.915	331.975	331.971
GPSA25S	332.413	332.225	332.217
GPSA27S	341.160	341.503	341.502
GPSA29S	342.644	342.211	342.212
GPSA30S	345.019	345.351	345.346

Table 5 Statistics of the results of differences between known and estimated orthometric heights values for the models

	Model A	Model B
Number of Stations (n)	21	21
Maximum Value (m)	0.432	0.432
Minimum Value (m)	-0.343	-0.342
Mean (m)	-0.013	-0.012
Std. Deviation (S)(m)	0.175	0.173
RMSE (m)	0.171	0.169

2.7. Determination of goodness of fit

The coefficient of determination (R^2) was used to determine the goodness of fit of the two models, and it is expressed by Sen and Srivastava (1990) as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} V_i^2}{\sum_{i=1}^{n}(q_i - \overline{q}_i)^2} \quad (13)$$

Where $\sum_{i=1}^{n} V_i^2$ the sum of the squared residual is adjusted for each of the stations in the fit and $\sum_{i=1}^{n}(q_i - \overline{q}_i)^2$ is the sum of the squared differences between the original height misclosures $q_i = (h_i - H_i - N_i)$ and their mean value $\overline{q}_i$. These statistical measures R^2 takes values between 0 and 1, the closer R^2 is to 1, the better the fit to the observation's measurements. Since the coefficient of determination (R^2) for each of the models are approaching the value of 1, it indicates a near perfect fit of the models.

2.8. Hypothesis testing

In this study, t-distribution statistics was used to test whether there is any significant difference in the performance of the two corrector surface models based on the mean orthometric heights. The hypothesis testing is stated below.

H_0 = The mean H of model A is equal to the mean H of model B

H_1 = The mean H of model A is not equal to the mean H of model B

Decision rule is given as: if $t_{cal} > t_{tab}$ at 0.05 significant level, reject H_0 and accept H_1

$$t_{cal} = \frac{|\bar{X}_A - \bar{X}_B|}{S_{AB}\sqrt{(\frac{1}{n_A} + \frac{1}{n_B})}} \quad (14)$$

$$\text{Pooled estimates } (S_{AB}) = \sqrt{\frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2}{n_A + n_B - 2}} \quad (15)$$

From Table 4 showing the orthometric heights from the two models, we have t_{cal} = 0.018 and from t table at degrees of freedom = 40 and 95% critical/confidence level, t_{tab} = 2.021.

Since $t_{cal} < t_{tab}$ that is, 0.018 < 2.021 we accept H_0 which means there is no significant difference between the orthometric heights obtained from the two models

2.9. Contour generation from Model A and Model B

The orthometric heights obtained from both models, A and B were used to generate contours using Surfer 10 software and kriging interpolation method. The contour plot is shown in Figure 1, Figure 2, and Figure 3 below.

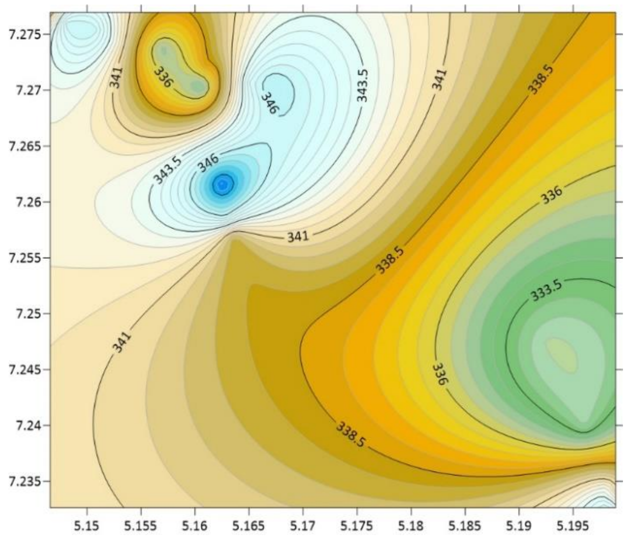

Figure 2.1. Contour plot generated from existing orthometric height

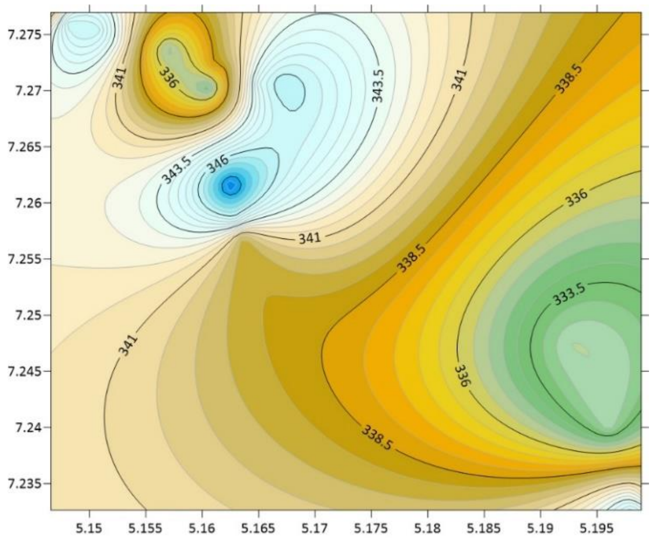

Figure 2.2. Contour plot generated from Model A

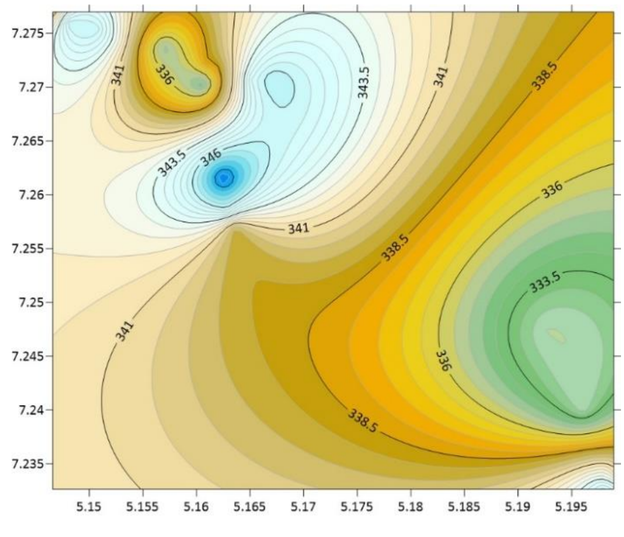

Figure 2.3. Contour plot generated from Model B

3. Results and Discussion

The seven and eight parameters datum shift transformation and their estimates are determined using the least squares method and the values of the parameters of these models are presented. The numerical results for the two models, as shown in Tables 2 and 3, revealed that the eight-parameter model gave the best fit, with a minimum standard deviation of 0.1563m compared to the seven-parameter model that gave 0.1568m. However, it should be well-known that the parameters from such a "a datum shift transformation" do not represent the true datum shift parameters (translations, rotations, and scale) because other long-wavelength errors inherent in the data (such as those in the height anomalies) will be interpreted as tilts and be absorbed by the parameters to some degree. Coefficient of determination was used to determine the goodness of fit of the two models, for which values closer to 1 were obtained which indicates a near-perfect fit of the models. For the minimization of residuals, both the seven-parameter and eight-parameter model were used successfully to absorb the datum inconsistencies between the height data and GPS, levelling and wavelength geoid errors and Root Mean Square Error (RMSE) was used to assess the performance of the models. In this study, the overall best agreement of 0.169m between the combined gravity field model EGM08 and GPS/levelling heights, was achieved when we used the eight-parameter model compared to 0.171m for the seven-parameter model. The use of the eight-parameter model slightly improves the residuals when compared to the seven-parameter model.

The result of the t-test statistics for comparison of the two models and the hypothesis test also showed the acceptance of the null hypothesis H_0 which indicates that there is no significant difference between the means of the models. This may be taken as validation that the two models successfully manage to absorb the datum inconsistencies between the height data, GPS, levelling, and long-wavelength geoid errors within the study area.

4. Conclusions

The paper has assessed the performances of two corrector surface models for orthometric height determination from GPS/levelling and the Global Gravity Model (EGM08). In this study, satellite gravimetry based EGM08 solutions for twenty-one (21) control points were fitted into GPS/levelling within the study area. Model A (seven-parameter) and Model B (eight-parameter) corrective surface models were incorporated to minimize the effect of datum inconsistencies and systematic effects in the combination of the data sets. The results obtained from the study show that model B gives a better result, with an RMSE value of 0.169 m, than model A. Considering the results obtained from the analysis, the use of a corrective surface model (such as Model B) to combine GPS/levelling with the gravity model EGM08 significantly improved the determination of orthometric heights through GPS observation in the study area. Thus, the incorporation of a corrective surface model into GPS measurements combined with an accurate geoid model can be used as a possible approach for direct conversion of ellipsoidal height to orthometric height.

5. Conflict of Interest

The authors declare no competing interest.

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