

# Ranking of Waste Management Options Under Conditions of Possibilistic Uncertainty Using Fuzzy SAW

**Raymond Girard R. Tan**

Chemical Engineering Department  
De La Salle University–Manila  
2401 Taft Ave., Manila 1004 PHILIPPINES  
Email: tanr\_a@dlsu.edu.ph

Integrated waste management involves the use of appropriate techniques ranging from pollution prevention/cleaner production (P2/CP) to conventional end-of-pipe controls. Design or retrofit of process plants usually entails selection of an optimal waste management measure from a number of alternatives. The selection process involves consideration of multiple criteria and data uncertainty, the latter being arguably possibilistic (fuzzy) rather than probabilistic (random) in nature. A fuzzy simple additive weighting (SAW) algorithm is proposed for such problems and demonstrated on a case study. The principal feature of the techniques shown is the retention of fuzzy confidence levels during the assessment of different technological options.

**Keywords:** Cleaner production, fuzzy numbers, pollution control, pollution prevention, and possibility theory.

## INTRODUCTION

The different options for the management of wastes in the process industries can be classified under four broad categories:

1. *Reduction or elimination at source* – use of inherently clean production operations and feedstocks through design for environment (DfE) principles or plant retrofits using process integration techniques. This approach is also commonly known as *pollution prevention* (P2) or *cleaner production* (CP).
2. *Recycling and reuse* – open- or closed-loop options to maximize utilization of material or energy resources.

3. *Treatment* – use of end-of-pipe controls to convert pollutants into less harmful form.
4. *Disposal* – isolation of residues to minimize interaction with the environment.

Of these approaches the first alternative is generally regarded as the preferred option, with the remaining options becoming progressively less desirable (Crittenden and Kolaczowski 1995, Sharrat 1999). In practical terms, the choice of which technique to use is influenced by engineering and economic as well as environmental considerations.

Integrated waste management relies on the identification of the best available technology (BAT) or best practical environmental option

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(BPEO) from a range of competing alternatives. Different decision aids have been used in the selection process, including AHP (Pineda-Henson et al. 2002), PROMETHEE (Le Teno and Mareschal 1998, Geldermann et al. 2000), and expert systems (Culaba and Purvis 1999).

Selection involves consideration of multiple criteria as well as data uncertainty, the latter being arguably possibilistic or fuzzy, rather than probabilistic, in nature. A simple multiple-attribute decision-making (MADM) technique is developed here for purposes of identifying optimal technologies under such ambiguous conditions.

**POSSIBILISTIC UNCERTAINTY AND FUZZY NUMBERS**

Probability theory is the most commonly used model of data uncertainty. *Classical probability* deals with quantifying tendencies of random events based on the frequency of occurrence of different outcomes. However, as explained by Tan et al. (2002), this theory is inappropriate for describing uncertainty arising from vagueness, subjectivity, or incomplete information. This form of uncertainty is fuzzy or possibilistic in nature and is best dealt with using fuzzy mathematics (Dubois and Prade 1988).

Imprecise quantities can be represented using fuzzy numbers (Kaufmann and Gupta 1991, Moore and Lodwick 2003). The membership function of a fuzzy number is also referred to as its *possibility distribution*. Generally, distributions represent the subjective degree of belief, or plausibility, of a range of values and, thus, unlike probability distributions, do not necessarily result from distinct mathematical rules. However, stylized triangular or trapezoidal distributions are often employed for simplicity (Mauris et al. 2001).

As an example, a fuzzy number with a trapezoidal possibility distribution is shown in Figure 1. The interval [2, 3] is called the *kernel* of the fuzzy number, and represents the most plausible range of values. This range is assigned a possibility level of 1. The interval [0.5, 4] represents the range of all marginally plausible values with nonzero possibility, and is called the *support* of the fuzzy number. For simplicity, trapezoidal fuzzy numbers are denoted here by

the extremes of the kernel and support, so that the number shown is (0.5, 2, 3, 4).

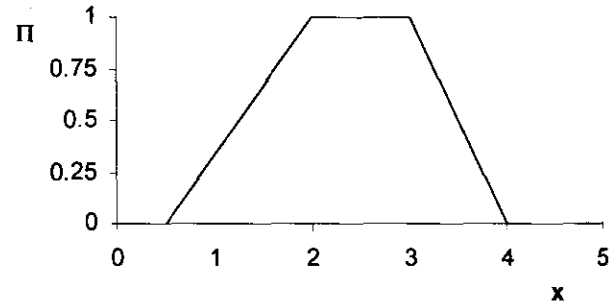


Figure 1. The Trapezoidal Fuzzy Number (0.5, 2, 3, 4)

**FUZZY SAW**

*Simple additive weighting* (SAW) is an MADM procedure that uses the weighted average of normalized scores in the different decision criteria to rank specified alternatives. It is widely used in different decision domains because of its simplicity. Here the basic SAW procedure is modified by using trapezoidal fuzzy numbers to represent imprecise scores and weights. Calculations are carried out using fuzzy arithmetic (Kaufmann and Gupta 1991, Moore and Lodwick 2003). The procedure involves calculating the total environmental impact rating of each alternative using Eq. (1):

$$(z_1, z_2, z_3, z_4)_j = \sum_i (w_1, w_2, w_3, w_4)_i \times \{(y_1, y_2, y_3, y_4)_{ij} \times [\max_j(y_4)_{ij}]^{-1}\} \quad (1)$$

where:

$(z_1, z_2, z_3, z_4)_j$  = fuzzy total environmental impact of option (j)

$(w_1, w_2, w_3, w_4)_i$  = fuzzy weight of environmental impact category (i)

$(y_1, y_2, y_3, y_4)_{ij}$  = fuzzy environmental impact of option (j) for category (i)

$\max_j(y_4)_{ij}$  = negative ideal solution for impact category (i)

A preliminary ranking of the alternatives can be done using the *defuzzified*, or "average," total scores. Eq. (2) uses the center of maximum (COM) to determine the crisp scores:

$$z_{\text{COM},j} = [z_{2,j} + z_{3,j}]/2 \quad (2)$$

where:

$z_{\text{COM},j}$  = defuzzified total environmental impact of option ( $j$ )

$z_{2,j}$  = lower bound of kernel of fuzzy total impact of option ( $j$ )

$z_{3,j}$  = upper bound of kernel of fuzzy total impact of option ( $j$ )

Different variants of fuzzy SAW have been in use for some time (Chen and Hwang 1992). However, there has been considerable disagreement as to how to establish outranking relationships among fuzzy aggregate scores, not only for SAW but also for related similar MADM methods (Stansbury et al. 1992, Geldermann et al. 2000, Chu 2002). Ranking based on crisp total scores disregards the information available with regard to the fuzzy uncertainty margins in the data. Hence, a more complete analysis should consider these margins and the associated confidence levels (Tan and Culaba 2001, Tan 2002, Tan et al. 2003). This is achieved by using Eq. (3) to determine the fuzzy indifference index for each pair of alternatives:

$$I(a, b) = \max \{ \min [\mu_{z_a}(x), \mu_{z_b}(x)] \} \quad (3)$$

where:

$I(a, b)$  = fuzzy indifference index between options ( $a$ ) and ( $b$ )

$\mu_{z_a}(x)$  = membership function of  $(z_1, z_2, z_3, z_4)_a$

$\mu_{z_b}(x)$  = membership function of  $(z_1, z_2, z_3, z_4)_b$

The indifference index falls in the interval [0, 1] and quantifies the extent to which the fuzzy scores overlap. Confidence in asserting the superiority of one alternative over another

increases as the indifference index decreases. It is possible to assign threshold values, in a manner analogous to statistical tests of hypothesis. Linguistic equivalents to different threshold values have been proposed by Tan (2002).

## CASE STUDY

The fuzzy SAW algorithm is illustrated using data from the case study of Geldermann et al. (2000). The problem involves identifying the best option for the treatment of waste gases generated by the ore sintering process in an integrated iron works, and was originally solved using fuzzy PROMETHEE.

The PROMETHEE method is an alternative to SAW for selection and ranking problems, but requires somewhat more complex computations to implement. Full details can be found in Geldermann et al. (2000). The alternatives to be evaluated are listed in Table 1.

**Table 1. Sintering Plant Air Emissions Control Options**

Option	Description
A	Electrostatic precipitator
B	Fabric filter with electrostatic precipitator
C	Cyclone
D	Wet scrubber

**Table 2. Environmental Impact Categories and Units**

Criterion	Environmental Impact Category	Unit	Fuzzy Weight
1	Photochemical Smog Formation	g C <sub>2</sub> H <sub>4</sub> equivalent	(2, 3, 3, 4)
2	Eutrophication	g PO <sub>4</sub> <sup>3-</sup> equivalent	(2, 3, 3, 4)
3	Acidification	kg SO <sub>2</sub> equivalent	(2, 3, 3, 4)
4	Human Toxicity	10 <sup>6</sup> m <sup>3</sup> (air)	(4, 5, 5, 6)
5	Ecotoxicity (Air)	10 <sup>6</sup> m <sup>3</sup> (air)	(2, 3, 3, 4)
6	Ecotoxicity (Water)	l (water)	(1, 2, 2, 3)
7	Hazardous Waste	kg	(1, 2, 2, 3)
8	Direct Fossil Energy Usage	GJ	(2, 3, 3, 4)
9	Electricity Usage	MJ	(2, 3, 3, 4)

Although these options represent air pollution control systems, an integrated or holistic evaluation scheme that takes into account a broader range of environmental flows and impacts is employed. These evaluation criteria are shown in Table 2. The fuzzy weights are based on the scale used in the original case study. In practice, these weights are problem-specific and are influenced by both company policies and government regulations. The COMs of the fuzzy weights are rated on a 5-point scale depending on the subjectively determined importance of each environmental category to the decision process, as shown in Figure 2.

The magnitudes of the environmental effects are expressed relative to production volume to

allow for uniform comparison. The basis used is 1 ton of sinter. Table 3 shows the numerical estimates of the impacts expressed as trapezoidal fuzzy numbers. These values were computed with Eq. (1), using fuzzy arithmetic implemented through alpha-cuts. Details of the algorithm are described by Kaufmann and Gupta (1991).

The fuzzy total environmental impacts of the four alternatives are shown in Figure 3. These fuzzy scores are dimensionless as a result of Eq. (1). Normalization into dimensionless form allows the aggregation of environmental impacts which are not directly comparable in the original units of measure. It must be emphasized, however, that these dimensionless aggregate indexes represent relative rather than absolute environmental

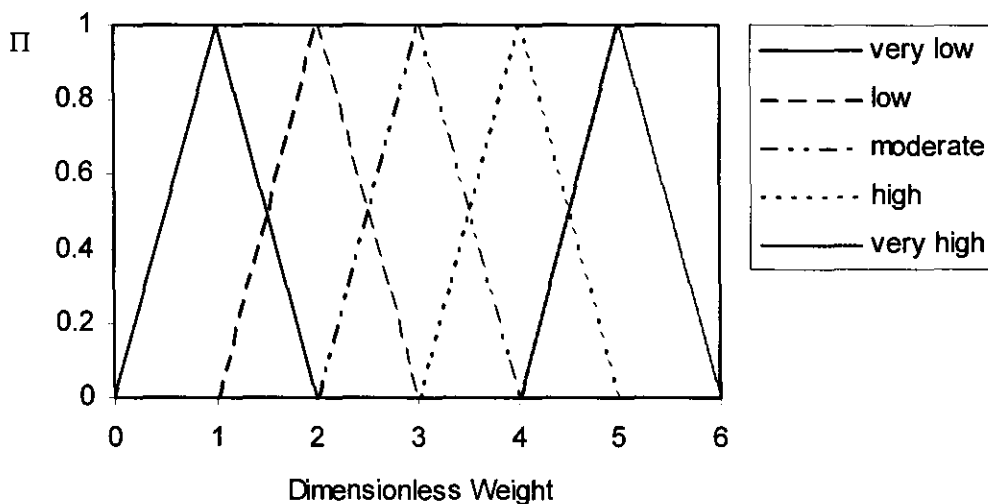
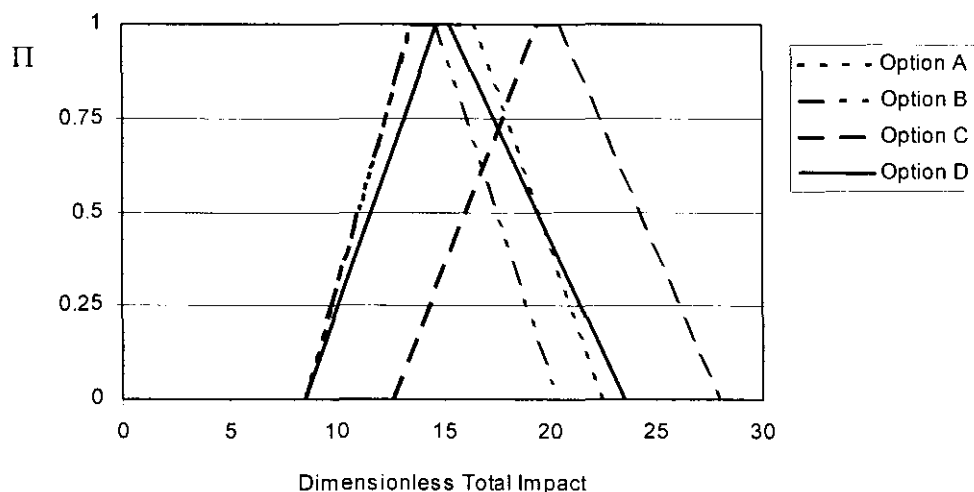


Figure 2. Subjective Weights and Fuzzy Numerical Equivalents (Geldermann et al. 2000)

Table 3. Environmental Flows and Impacts of Options per Ton of Sinter

Criterion	A	B	C	D
1	(31, 34, 37, 41)	(18, 19, 20, 22)	(100, 110, 130, 140)	(9, 10, 11, 12)
2	(46, 50, 55, 60)	(65, 68, 70, 76)	(57, 63, 65, 73)	(48, 51, 52, 56)
3	(1, 1.1, 1.2, 1.4)	(1.5, 1.7, 1.8, 2)	(1.5, 1.7, 1.8, 2.2)	(1, 1.1, 1.2, 1.6)
4	(110, 120, 190, 210)	(36, 40, 50, 55)	(180, 190, 200, 210)	(36, 40, 50, 55)
5	(20, 25, 35, 40)	(38, 40, 45, 48)	(53, 58, 60, 65)	(26, 28, 33, 35)
6	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(.2, .21, .22, .25)
7	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(.13, .15, .15, .19)
8	(1.65, 1.7, 1.7, 1.75)	(1.51, 1.56, 1.56, 1.61)	(1.6, 1.65, 1.65, 1.7)	(1.55, 1.6, 1.6, 1.65)
9	(355, 395, 395, 435)	(385, 425, 425, 465)	(305, 345, 345, 385)	(370, 410, 410, 450)



**Figure 3. Fuzzy Environmental Impacts of Pollution Control Options**

impacts. These are meaningful only in the context of comparisons among the four predefined alternatives. The addition of a fifth alternative, for instance, can change these impact values. Visual inspection of Figure 2 reveals that the total impacts of options A, B, and D are very similar, as indicated by the overlap in their possibility distributions. The aggregate impact of option C is distinctly higher than those of the other three.

**Table 4. Crisp Ranking of Options**

Option	Dimensionless Crisp Total Impact	Rank
A	14.9	2
B	14.0	1
C	19.9	4
D	15.0	3

If the comparison of the four alternatives is based on the defuzzified or “average” impact scores, the ranking shown in Table 4 will result. The use of crisp scores in this manner, however, defeats the purpose of having fuzzy quantities in the first place, since it ignores the effect of data uncertainty on the decision-making process. This point is the weakness of the methods developed and used by Stansbury et al. (1992), Geldermann et al. (2000), and Chu (2002).

For effective decision support, two levels of information must be preserved by the MADM technique: first, the estimates of the magnitude of the environmental impacts; and, second, the

associated *uncertainty margins*, or “confidence levels,” of these estimates (Tan et al. 2003). This principle underlies conventional statistical tests but applies equally well to the comparison of fuzzy numbers.

Uncertainty margins imply a threshold of indifference, which must be exceeded for superiority between any two alternatives to be definitively established. For example, fuzzy indifference indexes for the case study are shown in Table 5a. The values close to 1 indicate strong similarity between two alternatives. The indifference matrix may be defuzzified by assigning a threshold value for indifference. This process results in a binary state matrix that indicates a value of 0 for dominance and 1 for indifference.

**Table 5a. Fuzzy Indifference Matrix**

	A	B	C	D
A	1	1	0.77	1
B	1	1	0.6	0.98
C	0.77	0.6	1	0.72
D	1	0.98	0.72	1

Table 5b shows the defuzzified matrix for a threshold of 0.8. The matrix indicates that A, B, and D are not significantly differentiated, while C is definitively different from the other options. This result confirms the initial impressions derived from a cursory glance at Figure 2.

**Table 5b. Defuzzified Indifference Matrix**  
( $\alpha = 0.8$ )

	A	B	C	D
A	1	1	0	1
B	1	1	0	1
C	0	0	1	0
D	1	1	0	1

Adjusting the significance threshold further, as shown in Table 5c, will eventually result in the alternatives being deemed virtually equivalent.

**Table 5c. Defuzzified Indifference Matrix**  
( $\alpha=0.6$ )

	A	B	C	D
A	1	1	1	1
B	1	1	1	1
C	1	1	1	1
D	1	1	1	1

Hence, the key to effective use of this algorithm lies in selecting the proper threshold level. At the very least, the threshold used must be consistent with the linguistic framework or the scale used to deduce the possibility distributions of the input data.

Tables 5b and 5c, thus, indicate how the final decision on the degree of similarity changes as the threshold value is adjusted. Linguistic interpretations of numerical degrees of indifference (or dominance) have also been proposed (Tan 2002).

**CONCLUSIONS**

The use of fuzzy SAW for ranking waste management options has been demonstrated. Although fuzzy MADM techniques, including SAW, have been used for some time, the variant shown here differs in the use of thresholds of indifference to determine whether apparent differences in total environmental impacts are significant in light of the uncertainty contained in the input data used. This concept is analogous to the use of levels of significance in standard statistical tests of hypotheses.

The technique used here is promising in its simplicity and its ability to incorporate

fuzzy or possibilistic uncertainty in the aggregation and ranking procedures.

Such uncertainties arise from a variety of scenarios. For instance, comprehensive data is often not available for new or emerging technologies, thus making it necessary to use subjective expert estimates. When information is available, it may be incomplete, outdated, or otherwise not fully representative of the decision problem, as when data from another geographical region is used as surrogate in the absence of data for the actual plant site. Uncertainties are also present in the inherently subjective exercise of assigning weights or values to different environmental impact criteria.

The principal difficulty in the use of this technique is the ambiguity encountered in the interpretation of fuzzy uncertainty. Possibility levels are inherently subjective and are best interpreted in the context of linguistic expressions of belief or plausibility. This vagueness often seems to lead to confusion especially since the default model of uncertainty used by most people is the classical probability theory.

It must be noted that the practice of assigning a numerical index to a degree of belief is not fundamentally different from the Bayesian interpretation of subjective probabilities. Ultimately, the main benefit of the use of possibility theory in fuzzy SAW is that the decision-making process becomes more rational and self-consistent; however, in practice, the element of human subjectivity cannot be eliminated.

**NOMENCLATURE**

BAT	Best available technology
BPEO	Best practical environmental option
COM	Center of maximum
CP	Cleaner production
DfE	Design for environment
MADM	Multiple attribute decision-making
P2	Pollution prevention
SAW	Simple additive weighting

## REFERENCES

- Chen, S. J., and Hwang, C. L. (1992). *Fuzzy multiple attribute decision-making*, Springer-Verlag, Berlin.
- Chu, T. C. (2002). "Selecting plant location via a fuzzy TOPSIS approach," *Int. J. Adv. Manuf. Techn.*, 20, 859–64.
- Crittenden, B., and Kolaczowski, S. (1995). *Waste minimisation. A practical guide*, Institute of Chemical Engineers (IChemE), United Kingdom.
- Culaba, A. B., and Purvis, M. R. I. (1999). "A methodology for the life-cycle and sustainability analysis of manufacturing processes," *J. Clean. Prod.*, 7, 435–45.
- Dubois, D., and Prade, H. (1988). *Possibility theory: An approach to the computerized processing of uncertainty*, Plenum Press, New York.
- Geldermann, J., Spengler, T., and Rentz, O. (2000). "Fuzzy outranking for environmental assessment. Case study: Iron and steel making industry," *Fuzzy Set. Syst.*, 115, 45–65.
- Kaufmann, A., and Gupta, M. M. (1991). *Introduction to fuzzy arithmetic: Theory and applications*, International Thomson Computer Press, London.
- Le Teno, J. F., and Mareschal, B. (1998). "An interval version of PROMETHEE for the comparison of building products' design with ill-defined data on environmental quality," *Eur. J. Oper. Res.*, 109, 522–29.
- Mauris, G., Lasserre, V., and Foulloy, L. (2001). "A fuzzy approach for the expression of uncertainty in measurement," *Measurement*, 29, 165–77.
- Moore, R., and Lodwick, W. (2003). "Interval analysis and fuzzy set theory," *Fuzzy Set. Syst.*, 135, 5–9.
- Pineda-Henson, R., Culaba, A. B., and Mendoza, G. A. (2002). "Evaluating environmental performance of pulp and paper manufacturing using the analytic hierarchy process and life cycle assessment," *Journal of Industrial Ecology*, 6, 15–29.
- Sharratt, P. (1999). "Environmental criteria in design," *Comput. Chem. Eng.*, 23, 1469–75.
- Stansbury, J., Bogardi, I., Lee, Y. W., and Woldt, W. (1992). "Multiobjective decision-making under uncertainty," In: A. Goicochea, L. Duckstein, and S. Zionts, eds., "Multiple criteria decision-making," *Proceedings of the 9<sup>th</sup> International Conference—Theory and Applications in Business, Industry, and Government*, Springer-Verlag, Berlin.
- Tan, R. R. (2002). "Streamlined environmental life-cycle assessment using fuzzy semiquantitative evaluation matrices," *Proceedings of the 3<sup>rd</sup> Pacific Asia Conference on Mechanical Engineering*, Manila, Philippines.
- Tan, R. R., and Culaba, A. B. (2001). "Life cycle impact assessment using possibilistic compromise programming," *Proceedings of the 50<sup>th</sup> National Convention of the Philippine Association for the Advancement of Science*, Manila, Philippines.
- Tan, R. R., Culaba, A. B., and Purvis, M. R. I. (2002). "Application of possibility theory in the life cycle inventory assessment of biofuels," *International Journal of Energy Research*, 26, 737–45.
- Tan, R. R., Culaba, A. B., and Purvis, M. R. I. (2003). "Development of a life-cycle model using possibilistic uncertainty propagation and compromise programming for the evaluation of alternative motor vehicle fuels," *Proceedings of the First International Conference on Humanoid, Nanotechnology, Information Technology, Communications and Control, Environment and Management*, Manila, Philippines.