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Auto Regressive eXogenous (ARX) System Identification of Batch Milk Cooling Process

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Abstract. The dynamic model of the milk cooling process from 36°C to 4°C using chilled water available at 2°C has been carried out. The cooling water temperature is kept constant by using a refrigeration unit. The process being studied was a Packo brand milk cooling tank belonging to KUD SAE Pujon (Malang - Indonesia). A fundamental heat balance method was used to derive the model, leading to a first-order transfer function process. For a 2 hours cooling process then, the gain and time constant values are 1.00 and 42.3548 mins respectively, or $G(s) = \frac{1}{42.3548s+1}$ (first order process). Deriving system transfer function through a mechanistic model is considered difficult; therefore, in this paper, we explored process identification via Auto Regressive eXogenous (ARX). Transient simulations could then be performed to identify the dynamic behavior of the cooling process. The system was then identified using several orders of the Auto Regressive eXogenous (ARX) model, and then the results were re-tested on different forms of perturbations and obtained quite accurate results. The transfer function identified through the ARX111 is $G(s) = \frac{1}{42.3729s+1}$ (first order process), while via ARX441, the 5th order process was obtained: $G(s) = \frac{0.02361s^4+0.000371s^3+0.2331s^2+9.27\times10^{-7}s+0.0005826}{s^5+0.03932s^4+9.873s^3+0.2331s^2+0.02468s+0.0005826}$. These models particularly useful for process control design and analysis.

Keywords: Dynamic Study, Milk Cooling, Simulation, Process Identification

INTRODUCTION

The short shelf-life of raw milk is a concern for the dairy sector (Zacharski et al., 2021). Rapid cooling of raw milk temperature from the initial temperature (about 36°C) is widely used to preserve milk during transportation or storage in dairy cooperatives (Lee et al., 2016). When milk is stored above 30°C, it may spoil after 4 hours, depending on the initial total bacteria (Al-Shannaq et al., 2022). Storing at 20°C, 10°C, and 4°C partially inhibited bacterial growth for the first 8, 16, and 24 hours after milking. However, refrigeration of milk is a stabilizing treatment and does not improve milk quality or kill any bacteria; therefore, it must be combined with good herd management and hygiene standards (Torres-Toledo et al., 2018).

The milk cooling model is needed to design, train and control the milk cooling

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system (Bequette, 2019). There are 2 kinds of dynamic model approaches that are often used, namely the white-box model and the black-box model (Kaufhold et al., 2021).

The white box model is based on the laws of conservation of physics and chemistry, such as mass balance, component balance, momentum balance, and energy balance. They are also called the first principle or mechanistic model (Ochsenbein et al., 2019). This model provides an understanding of the process and explains the behavior of the process in terms of state variables and measured variables (Pauli et al., 2019). The state variable of the model is the variable whose rate of change is described by the law of conservation. These models can already be developed when the process does not yet exist. Dynamic equations are supplemented by algebraic equations explaining heat and mass transfer, kinetics, etc. (Cheak Theng Ee et al., 2021). Developing this model is considered time-consuming (Behnam et al., 2022).

Black box models or empirical models do not describe the physical phenomena of the process; they are based on input/output data and only describe the relationship between measuring input and output data from the process (Fung et al., 2021). This model is useful if limited time is available for model development and/or if there is not an adequate physical understanding of the process. Mathematical representations including time series models such as Auto Regressive Moving Average (ARMA), Auto Regressive eXogenous (ARX), Box and Jenkins models, recurrent neural network models, and recurrent fuzzy models (Chakraborty et al., 2022).

Many empirical methods exist in which the a priori model structure has to be selected and further model parameters identified by appropriate techniques. The method that can be used is the least squares method (Yin et al., 2022).

Suppose a process with input $u(k)$ and output $y(k)$ where k is a discrete time value. Assuming that the signal can be related to a linear process, we can write:

$$A(z^{-1})y(k) = \frac{B(z^{-1})}{F(z^{-1})}u(k-p) + \frac{C(z^{-1})}{D(z^{-1})}e(k) \quad (1)$$

Where $z^{-1}$is the shift operator and is defined as the following:

$$y(k-1) = z^{-1}y(k) \quad (2)$$

While the polynomials A, B, C, D, and F are given as follows:

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{na} z^{-na} \quad (3)$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + \ldots + b_{nb} z^{-nb} \quad (4)$$
$$C(z^{-1}) = c_0 + c_1 z^{-1} + \ldots + c_{nc} z^{-nc} \quad (5)$$
$$D(z^{-1}) = d_0 + d_1 z^{-1} + \ldots + d_{nd} z^{-nd} \quad (6)$$
$$F(z^{-1}) = f_0 + f_1 z^{-1} + \ldots + f_{nf} z^{-nf} \quad (7)$$

Meanwhile, p is the sampling interval between the input and output of the process, and e is the modeling error (Roffel and Betlem, 2006).

In the literature (Cheng et al., 2021), the structure of the Auto Regressive eXogenous (ARX) model is as the following:

$$A(z^{-1})y(k) = B(z^{-1})u(k-p) + e(k) \quad (8)$$

Which indicates that $nc = nd = nf = 0$. Example:

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-3) + b_2 u(k-4) \quad (9)$$

The process identification strategy by measuring input-output is usually used in many situations where it is not necessary to achieve deep mathematical knowledge of the system being studied, but it is sufficient to study the dynamics of the system

(Bajarangbali et al., 2014, Dhanya Ram and Chidambaram, 2014).

This paper will identify back with certain perturbations from a predetermined process model, namely the batch process of cooling milk at KUD SAE Pujon (Malang) (Agustriyanto, 2019). The process will be identified with the ARX model, and the results will be compared with the model that has been obtained. Matlab and System Identification Toolbox (R2022a, n.d.) will be used in this research.

The model obtained is intended to achieve best practices in dairy processing. The characteristics of plants/milk processing installations that seek to achieve best practices are available in the literature (Penny Prasad et al., 2019), but the model plays an important role in the field of process control (Li et al., 2020).

MATERIALS AND METHODS

The system being studied was the milk cooling facilities belonging to Koperasi SAE Pujon, as shown in Figure 1. It is a rectangular milk cooling tank with a half-cylinder tank contained therein. The rectangular tank is used as a place for cooling water, while the half-cylinder tank is used as a container for milk. The scheme of the milk cooling tank is shown in Figure 2. Table 1 shows batch and parameter data for the simulation (Agustriyanto et al., 2022)

The tank capacity (*V*) is 2500 L; therefore, the mass of milk in the tank can be calculated since the milk density is known. The volume of chilled water is unknown, but its temperature is kept constant at 2°C (T_1) by using refrigerant.

Fig. 1: Packo milk cooling tank (Courtesy of Koperasi SAE Pujon)

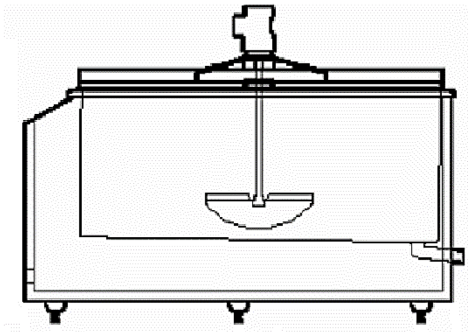

Fig. 2: Stirred milk cooling tank system seen for the side (Courtesy of Koperasi SAE Pujon)

Table 1. Batch and Parameter Data

Symbol	Value	Unit
V	2500	L
T_i	36	˚C
T_o	4	˚C
T_1	2	˚C
C_p	3.93	kJ/kg.K
ρ	1027	kg/m^3
m	2567.5	kg
U	274.4613	kJ/min.m^2.°C
A	0.8680	m^2
Q	2690.74	kJ/min

If we set the cooling time equal to 2 hours, then the total heat that must be released from milk to cooling water can be calculated as follow:

$$Q = m.C_p.\Delta T \quad (10)$$

$$Q = 2567.5 \times 3.93 \times (36 - 4)$$

$\mathrm{Q} = 322{,}888.8\ kJ/2h$

$\mathrm{Q} = 2690.74\ kJ/min$

Heat transfer area (A) is the area of a half-cylinder that can be calculated mathematically, where:

$r = 0.3485\ m\ and\ t = 1.238\ m$

$$\mathrm{A} = \frac{1}{2}\pi r(r+t) = 0.868\ m^2 \quad (11)$$

Log mean temperature difference (LMTD) for parallel flow can then be calculated (Jouhara et al., 2021; Li et al., 2017):

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \quad (12)$$

$$\Delta T_{LMTD} = \frac{(36-2)-(4-2)}{ln\left(\frac{36-2}{4-2}\right)} = 11.2946$$

Since: $Q = U.A.\Delta T_{LMTD}$ (13)

Then: $U = 274.4613\ kJ/(min.m^2.°C)$

The batch milk cooling process can be modeled based on energy balance as the following (Coughanowr and LeBlanc, 2009):

$$Q_{in} - Q_{out} = Accumulation \quad (14)$$

When $Q_{in} = 0$, then

$$0 = U.A.\Delta T = \frac{dQ}{dt}$$

$$-U.A.(T_o - T_1) = m.C_{p.}.\frac{dT_o}{dt}$$

$$U.A.(T_1 - T_o) = m.C_{p.}.\frac{dT_o}{dt} \quad (15)$$

Steady State Energy Balance

$$U.A.(T_{1s} - T_{os}) = 0 \quad (16)$$

Equation (15) – (16):

$$U.A\big((T_1 - T_{1s}) - (T_o - T_{os})\big) = m.C_{p.}.\frac{dT_o}{dt} \quad (17)$$

Taking Laplace Transform of both sides in Equation (17):

$$UA(T_1(s) - T_o(s)) = mC_p(sT_o(s) - T_o(0)) \quad (18)$$

When $T_o(0) = 0$, then:

$UA(T_1(s) - T_o(s)) = mC_p sT_o(s)$, or

$$\frac{T_o(s)}{T_1(s)} = \frac{1}{\frac{mC_p}{UA}s+1} \quad (19)$$

The resulting Equation (19) is a first-order process transfer function with:

$$Gain = K_p = 1 \quad (20)$$

$$Time\ constant = \tau_p = \frac{mC_p}{UA} \quad (21)$$

Hence:

$$\frac{T_o(s)}{T_1(s)} = \frac{1}{42.3548s+1} \quad (22)$$

While Agustriyanto et al. (2022) explored the dynamic behavior based on this mechanistic model, here we would like to extend the process identification via the Auto Regressive eXogenous (ARX) model.

Transient simulations could then be performed to identify the dynamic behavior of the cooling process (Figure 3). The system was then identified using several orders of the ARX model, and the results were re-tested on different forms of perturbations.

RESULTS AND DISCUSSION

The milk cooling process in Cooperatives (Koperasi Unit Desa in Indonesian) is generally carried out as a batch process. The batch process only depends on time. When the milk temperature has reached the expected, then the milk is released. Assuming that the overall heat transfer coefficient (*U*) is constant, the result of the mathematical model is shown in Equation (22).

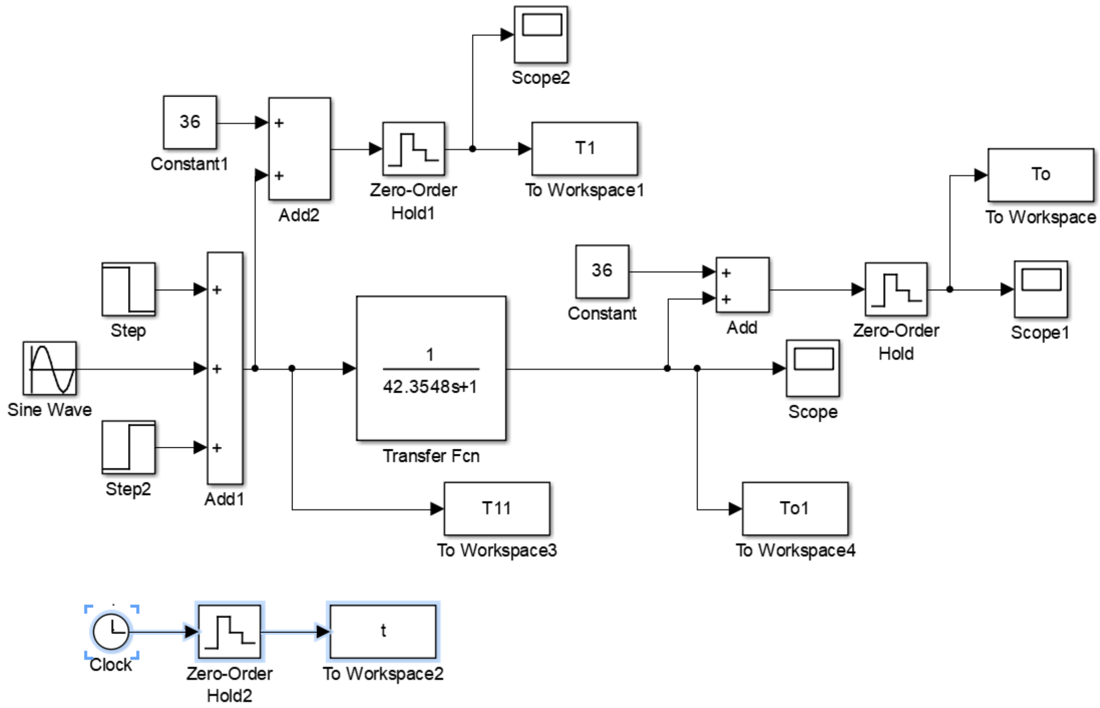

Fig. 3: Simulink block diagram

From the mathematical model of the batch system above, the simulation results are shown in Figure 4. Based on this figure, it can be seen that the batch system only affects cooling time. The milk temperature comes in at 36°C and reaches 4°C with a cooling time of 120 minutes (2 hours).

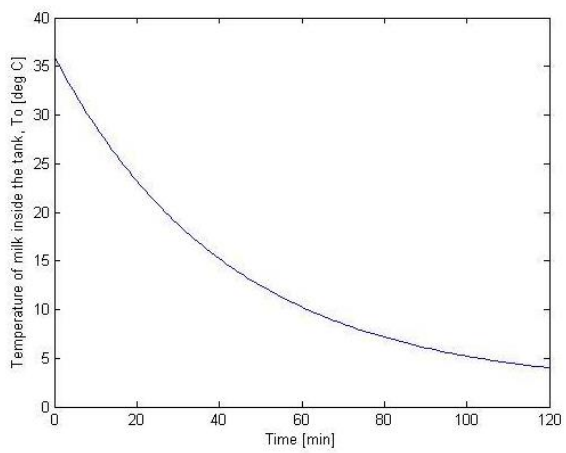

Fig. 4: Milk temperature profile

Since the gain is 1 while the time constant is 42.3548 minutes, the process will reach 2°C at more than 4 times the time constant. The time constant is the time required for the first-order process to reach 63.2% to the new steady state value when a step disturbance is given to the input variable.

Equation (22) was then simulated using Simulink and given the disturbance at the cooling water temperature. The shape of the disturbance function is shown in Figure 5, while the milk temperature profile is shown in Figure 6.

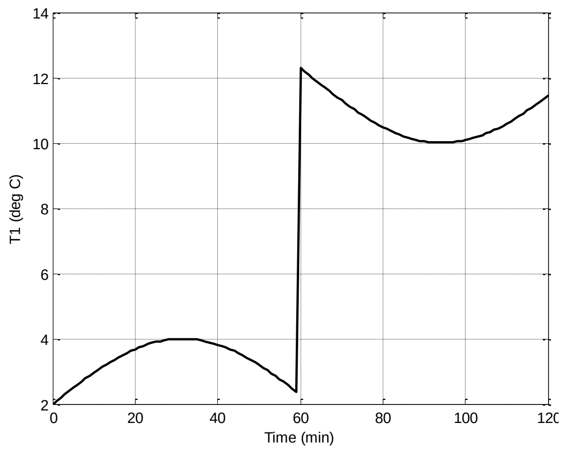

Fig. 5: Chilled water temperature profile

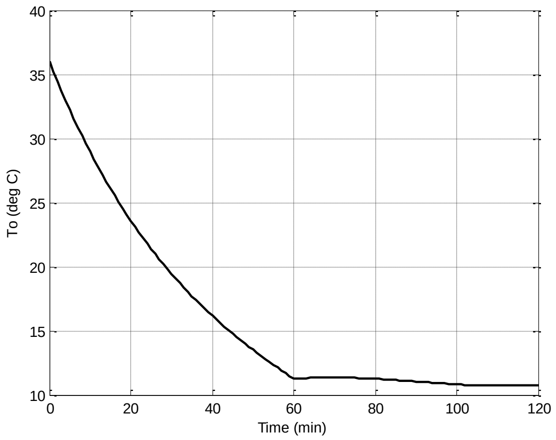

Fig. 6: Milk temperature profile

The data were then identified using the System Identification Toolbox in Matlab. The identification results are shown in Table 2. So that the ARX transfer function model obtained is:

$$G(z) = \frac{To(z)}{T_1(z)} = \frac{0.0233z^{-1} - 0.02336z^{-2} - 0.02291z^{-3} + 0.02315z^{-4}}{1 - 1.982z^{-1} + 1.951z^{-3} - 0.969z^{-4}} \quad (23)$$

Table 2. The Results of ARX Model

Coeff	Order 4 4 1				
A	1	-1.9820	0	1.9511	-0.0960
B	0	0.0233	-0.0235	-0.0229	0.0232
C			1		
D			1		
F			1		

This discrete transfer function could then be converted to a continuous transfer function with the d2c command to obtain:

$$G(s) = \frac{To(s)}{T_1(s)} = \frac{0.02361s^4 + 0.000371s^3 + 0.2331s^2 + 9.27\times10^{-7}s + 0.0005826}{s^5 + 0.03932s^4 + 9.873s^3 + 0.2331s^2 + 0.02468s + 0.0005826} \quad (24)$$

The next stage is to test the suitability of the model obtained with a certain another form of perturbations (as shown in Figure 7) of chilled water temperature. Figure 8 shows the response plot for the temperature of the milk inside the tank; the simulation results of Equation (24) (the identification model) are compared with Equation (22) (the derivation model).

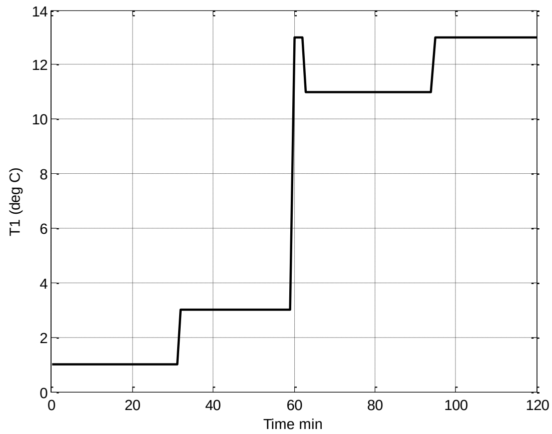

Fig. 7: Chilled water temperature profile

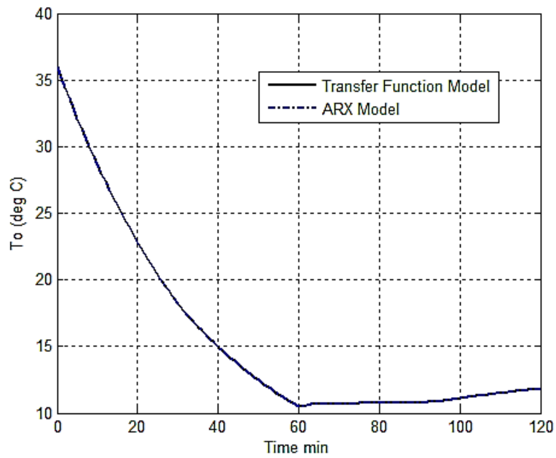

Fig. 8: Milk temperature profile

It appears that the model identified by the ARX method gives similar results as the model derived from the derivation, so it can be said that this model is quite accurate.

However, this ARX method depends on the choice of its order. The ARX model in Equation (24) is a 5^{th}-order transfer function, which is too complicated for control tuning and stability analysis. Selecting order 1 1 1 (instead of 4 4 1) will give the ARX model as follows:

$$G(z) = \frac{T_o(z)}{T_1(z)} = \frac{0.02332z^{-1}}{1-0.9767z^{-1}} \quad (25)$$

Converting Equation (25) to continuous time, the following equation was obtained:

$$G(s) = \frac{T_o(s)}{T_1(s)} = \frac{0.0236}{s+0.0236} \quad (26)$$

or

$$G(s) = \frac{T_o(s)}{T_1(s)} = \frac{1}{42.3729s+1} \quad (27)$$

Note that equation 27 is very similar to equation 22. Therefore, it is no need to show its simulation results.

CONCLUSIONS

The dynamic model of the batch milk cooling process has been derived and simulated using Simulink. The overall heat transfer coefficient (*U*) was assumed constant and could be calculated as the batch process finished in 2 hours. The milk temperature profile shows reasonable dynamics during the process.

The dynamic model of the milk cooling batch process is:

$$G(s) = \frac{T_o(s)}{T_1(s)} = \frac{1}{42.3548s+1}$$

The model has been simulated with a certain form of disturbance from cooling water so that the transfer function is obtained/identified through the ARX441 method as follows:

$$G(s) = \frac{To(s)}{T_1(s)} = \frac{0.02361s^4 + 0.000371s^3 + 0.2331s^2 + 9.27\times10^{-7}s + 0.0005826}{s^5 + 0.03932s^4 + 9.873s^3 + 0.2331s^2 + 0.02468s + 0.0005826}$$

or

$$G(s) = \frac{T_o(s)}{T_1(s)} = \frac{1}{42.3729s+1}$$

which was obtained via ARX111.

Then this transfer function was tested when given different forms of perturbation, and it was found that the ARX441 model identified was quite accurate.

The ARX model used is a model with the order of 4 4 1 and 1 1 1. Order selection is one of the important factors so that the model obtained can represent the actual process. The mathematical model is particularly useful for control purposes.

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NOMENCLATURE

V	:	tank volume [L]
T_i	:	the temperature of inlet milk [°C]
T_o	:	the temperature of milk inside the tank [°C]
T_1	:	the temperature of chilled water [°C]
C_p	:	heat capacity of milk [kJ/kg.K]
ρ	:	the density of milk [kg/m^3]
m	:	mass of milk in the tank [kg]
U	:	Overall heat transfer coefficient [kJ/min.m^2.°C]
A	:	the surface area of the milk cooling tank [m^2]
Q	:	the heat released from milk to cooling water [kJ/min]

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