Gadjah Mada International Journal of Business
January-April 2005, Vol. 7, No. 1, pp. 15—39

TRADING BEHAVIOR AND ASSET PRICING UNDER HETEROGENEOUS EXPECTATIONS

R. Agus Sartono

This research models trading behavior and examines the impact of heterogeneous expectations on asset prices. We extend Kyle's (1985) one-period model to two-period model. The model shows that the informed trader takes into account not only the private information but also the pricing function. The price is an increasing function of the volatility of the asset value and decreasing in the volatility of uninformed traders' demand. The costly information acquisition has an impact on the optimum demand but it has no direct impact on the price.

We find the market depth is a linear function of the volatility of the uninformed traders and a weighted average of the total error variance of information. The depth is also decreasing in the volatility of the cash flow innovations. This argument is in line with the second finding, when the volatility of cash flow innovations increases, the value of risky asset becomes more volatile, and as a result the bigger are the advantages of having private information. Our research raises some questions for further investigation. We indirectly assume that the informed traders make a profit at the expense on the uninformed traders. The question is why the uninformed traders willing to face losses? What happen if there are n *informed traders who have diverse information?*

Keywords: asset price behavior; cash flow innovations; heterogeneous expectations; informed traders; market depth

Research Background

Homogeneous expectations, cost-free information and risk aversion are crucial assumptions underlying the capital asset pricing model, hereafter called CAPM. As the standard model of financial markets, the CAPM assumes homogeneous probability expectations:[1] individuals have the same information, interpret it in the same manner and draw the same conclusions. However, in the real market, even when individuals have the same information, they will most likely analyze and interpret it in different ways and come up with different conclusions. Empirical studies of the CAPM are typically characterized by the assumption of mean variance expectations derived from historical data and risk-averse traders. Roll (1977) raised some serious questions about the validity of the homogeneous expectation assumption and the testability of the CAPM. A valid test requires complete knowledge of the composition of the true market portfolio. This implies that all assets must be included in the test. Despite its many critics, the CAPM has drawn a great deal of attention from practitioners as well as academic communities within the last three decades. The CAPM has proved to be a preferable approach for pricing assets

The Efficient Market Hypothesis (EMH) has become another major debate subject among academics and financial professionals in the last three decades. The EMH states that a market is said to be informationally efficient if at any given time, security prices fully reflect all available information and the actual price of a security is a good estimate of its intrinsic value (Fama 1970). Thus, in an informationally efficient market it is impossible to make profits on the basis of public and private information. In reality, the markets are neither efficient nor completely inefficient. All markets are efficient to a certain extent and some more so than others.

The empirical studies indicate that there are quite a lot of anomalies such as size effect, end-of-the-year effect, price-to-book ratio effect, and price reversal that can not be justified by the EMH.[2] There is continuing disagree-

[1] The models of both William F. Sharpe (1964) and John Lintner (1965) assumed that agents agree on mean and variance and the only risk that should be compensated is the systematic risk which can not be eliminated by diversifying investment in the portfolio. Heterogeneous belief or divergence opinion is used with the same purpose as heterogeneous expectations. It captures how traders collect, analyze and interpret different information about fundamental value of assets differently.

[2] For more details, here is a short list of some anomalies: dividend cut and omissions (Michaely et al. 1995); stock repurchase (Lakonishok and Vermaelen 1990); stock return and weekend effect (French 1980); book to market value equity (Fama and French 1992); the January effect (Rozeff and Kinney 1976; and Keim 1983); size effect (Banz 1981, and Reinganum 1983); P/E effect (Basu 1977); price reversal (DeBondt and Thaler 1985, and Jegadesh and Titman 1993); and equity premium puzzle (Mehra and Prescott 1985).

ment as to whether the different anomalies indicate that the market is inefficient or whether the anomalies stem from a mathematical modeling problem, a sampling error, or misinterpretation and misuse by practitioners and researchers. Fama (1991) admits that zero information cost is a necessary condition for an efficient market. Since information is cost-free, traders are assumed to have the same information, and the price system leads to perfect aggregation of information. Whenever prices perfectly aggregate all information, private information is useless and traders could infer information from the prices.

If this is the case, there is no reason for traders to spend a single euro to get additional information. If all available information is reflected in the asset prices, there is no incentive for information acquisition. Grossman (1976) argues that the cost-free information is not only a sufficient, but also a necessary condition. In another paper, Grossman and Stigliz (1976, 1980) and Grossman (1978) argue that the capital market can not even begin to be informationally efficient if the acquisition of information is costly.

The main point of the argument is that under the costly information assumption, if prices fully reveal all information, traders may not be able to earn a return on their investment in information. If the price system is a perfect aggregator of information, it eliminates the private incentive to collect information. All traders prefer to obtain the available information at no cost by inferring it from the market prices rather than by acquiring it directly at cost. If there is no incentive for information gathering, the perfect competitive market will break down; there is no equilibrium since no one collects costly information. If no one collects the information, how can prices reflect the information; and the prices will not reveal all the information. Then some traders may realize that they would be better-off if they were better informed. In equilibrium, the price must contain noise, a disturbance that prevents traders from learning all the information from the price and provides incentive for information acquisition. Informed traders can then hide and maximize their advantage of having private information. Otherwise, the markets break down.

In a competitive market, if traders have heterogeneous information, then their demand for risky asset may reveal or partially reveal their information. As in Grossman's (1976) model with two types of risk-averse informed and uninformed traders, if the price is high then the uninformed can infer that this may be due to the informed traders having good news regarding the expected payoff of assets. But whenever the price is low, this implies that informed traders possibly just received bad news. Assuming that all traders are rational, price is a sufficient statistic for all the signals observed by traders. Knowing the equilibrium price is the same as observing all the information, and private information is useless. This argument enhances the paradox that price can incorporate the traders' information only if traders take it into account in determining their demand. Meanwhile, if they know the equilibrium price, they will disregard their information in favor of the equilibrium price.

Hellwig (1980) and Grossman and Stiglitz (1980) attempted to solve the asset pricing paradox by introducing the supply of risky assets and liquidity traders as random variables. Diamond and Verrecchia (1981) consider the random supply of risky assets as noise. If the supply is uncertain, then the high price could be due to favorable information or the low supply of risky assets. Price will no longer be perfectly informative, and traders should take into account their own information as well as the price. Kyle (1985) proved that when traders take into account the effect of their own demands on price, the price is not fully revealing. It thus provides incentive for costly information gathering even if traders are risk-neutral.

Purpose of the Study and Research Question

Investors or traders spend millions of euros to gain access to the increasingly sophisticated forecasting services of professional security analysts, who provide earnings forecasts and statistical analyzes on a firm-by-firm basis. An analysis can be yearly or quarterly, for an individual company or for an entire industry. Information is costly, and the main objective in acquiring costly information is to have better assessment of assets value. Traders gather, analyze and interpret information differently, so they end up with different beliefs regarding the future pay off or cash flow of assets. Different beliefs lead to different or heterogeneous expectations.

The purpose of this study is to analyze the impact of heterogeneous expectations on the asset price. Assuming that the market is somewhat inefficient and that the prices do not fully reflect all the available information, being better informed should yield a higher return. We will extend the one-period Kyle (1985) model to a two-period model in an attempt to find solutions for asymmetric information in asset pricing. We expect that our result will answer the Grossman Paradox to some extend. Using Kyle's (1985) framework, we will focus on the strategic action of an informed trader who has better information than uninformed traders. Information is costly and traders are assumed to be risk-neutral, so there will be no risk sharing problems among traders and there must be incentives for traders who choose to be informed.

Admati and Pfleiderer (1988) suggest there are informed traders, uninformed traders, and liquidity traders. The informed traders have private information about cash flow innovations, but they have different error variance of private information. The main intention is to see whether the diverse information that presumably leads to heterogeneous expectations has an impact on the optimum demand for risky assets. They focus on the dynamic strategic behavior of informed and liquidity traders. Informed traders have private information on the realization value of assets, but the liquidity traders do not have such information. The liquidity traders fall into two categories, discretionary liquidity traders, who use their own discretion in timing trades, and non-discretionary liquidity traders, who do not have discretion in timing.

The informed trader has lower error variance of private information than the uninformed trader. The informed traders are those who have special knowledge about cash flow innovations, and who spent a lot of effort and resources to gather and analyze information. On the other hand, the uninformed traders are those who have no special knowledge about cash flow innovations, and who make no effort to gather and analyze information. In our scenario, we simply assume that there are constant flows of uninformed traders coming into the market. Having a large number of uninformed traders in the market creates camouflage for informed traders' private information. Thus, when some uninformed traders leave the market, the other uninformed or liquidity traders just enter the market. That is true of the real world, where some numbers of rational traders leave the market and are replaced by other traders. Grossman (1976) and Kyle (1985) indirectly assumed that the uninformed traders can not buy the market portfolio. That is why they lose to trade with the informed traders. If the uninformed traders can buy the market portfolio, they are guaranteed to earn the average return.

In fact, traders do spend a lot of money to gather information. They do interpret and analyze in different ways, because they have different information processing capacity. As a result they have different beliefs and heterogeneous expectations regarding assets value. The main question then, does the private information have an impact on the equilibrium asset price? How the price behaves in the equilibrium using an extended two-period model? Do the heterogeneous expectations, as measured by different error variances of private information, have an impact on the demand for risky assets? These questions are at the center of this research. We intend to answer these questions by building a mathematical model.

Brief Overview of Previous Studies

Lintner (1969) raised the issue of heterogeneous expectations, and traders are assumed to be risk-averse. He proposed a model in which traders have a different assessment of expected price and covariance. When expected futures prices differ among traders, each trader will hold different optimal portfolios and different fractions of the portfolio. The pricing system takes the weighted average of traders' portfolio. The expected market prices are a complex weighted average of individuals' assessment. In a purely competitive market, the price system necessarily and automatically uses the ideal weight in combining the separate vector assessment of expected price of each individual into its composite market estimate price. The best composite estimate price, the market's composite risk aversion and its composite covariance assessment together determine the market prices. Thus, the pricing system aggregates the individuals' heterogeneous assessment of future price and covariance into a single market price.

It is Kyle (1985) who starts developing the asset price under asymmetric information. He assumes there is one risk-neutral insider or informed trader, market makers, and noise traders. The market maker knows only the sum of the demands of the informed and noise traders, but does not know them individually, and uses this information to set the market price and then take a position to clear the market. It is assumed that market making is a perfectly competitive profession, so that the market makers set the price such that, given the total order, their profit at the end of the period is expected to be zero. The price is a linear function of the total order flows. Kyle proved that insider, as an informed trader, makes a positive profit by exploiting his power monopoly optimally in a dynamic context, where noise trading provides camouflage, which cancels his trading from market makers. To simplify the analysis, the noise traders are assumed to be the same as uninformed noise traders.[3] In this model, the informed trader is assumed to be risk-neutral who maximize the expected profit. The price determined by the market maker is assumed to be equal to the expectation of the liquidation value condition on the market maker's information set.

Gorton and Pennacchi (1993) extended the one-period Kyle model to a two-period model. They focused on the behavior of uninformed traders in an attempt to minimize loss trading with informed traders. In the first period the company sells share in exchange for capital. Some uninformed traders face liquidity needs, become early consumers and have to trade, while the rest remain as late consumers. With the existence of forward markets, the early consumers can sell their share forward to fulfill their cash problem. The fully rational uninformed traders can create composite securities to minimize their loss. In other papers, Grossman (1976) and Grossman and Stiglitz (1980) showed how the heterogeneous information of different traders can be reflected on the equilibrium price of security. Using two types of traders who are informed and uninformed, they demonstrated how the

[3] It is not absolutely correct to assume that the uninformed traders trade based on the noise as in Black's (1986) concept. However, to simplify the research, we use the uninformed traders, liquidity traders and noise traders interchangeably.

price system transmits information from informed to uninformed traders. Nevertheless, if the price system aggregates perfectly private information, then it eliminates the incentive for traders to collect information.

In another attempt to model the costly information on asset price, Diamond and Verrecchia (1981) introduced noise by assuming that traders have an initial random endowment of the asset. If the supply to rational traders is not known with certainty, then a high price could be either the consequence of traders receiving favorable signals or the effect of low supply. Price is no longer perfectly informative about the traders' information set, and so each trader will base his belief about the likely outcome on both price and his own piece of information. Jackson (1991) analyzed the equilibrium, price formation and the value of private information. He showed that if the price formation process is modeled explicitly and traders are not price takers, then it is possible to have equilibrium with fully revealing prices and costly information acquisition. He used a similar model as Grossman (1976) with two types of traders but under the assumption of risk-neutral traders. He dropped the price taking assumption, because it is impossible to have a fully revealing price if information acquisition is costly. Equilibrium in costly information acquisition is ex ante Pareto inefficient, since resources are lost in the acquisition of information. In his setting, information is only important in determining the distribution of wealth among identical risk-neutral traders; therefore, it is not surprising that costly information acquisition is inefficient. He proved that if the price formation is explicit and traders influence the prices, then it is possible to have fully revealing prices.

Jackson and Peck (1997) examined price formation in a simple static model with asymmetric information and the advantage of being informed traders. They showed that with costly information acquisition, the price exhibits a "V" shape as a function of the cost of information. When all traders are either informed or uninformed, the price is highest, and lower when only some traders are informed. They built a model in which there is an infinite number of traders. Thus, the price formation is competitive and no single trader can influence the price. The rationale of the "V" shape of expected prices is straightforward; when the cost of information acquisition is low, everyone chooses to be informed; but as the information cost increases, the fraction of informed traders decreases. When the information cost exceeds the maximum potential benefit from being informed, no one chooses to be informed. The benefit of being informed at the beginning declines as the number of informed traders increases and at some point the benefit of being informed increases.

William (1977) introduced an alternative model in pricing capital assets under heterogeneous beliefs and risk-aversion. Using his model, he proved that beta is no longer a complete measure of risk. Figlewski (1982) proved that information level or diversity will affect investors' beliefs or expectations. Thus, different investors may acquire different information, process it in different ways and respond differently. In another study based on a computer simulation model, Schredelseker (1997) developed a model that indicates that information does have a negative value and does not supports the empirical evidence that investors are willing to spend a lot of money to get better information in advance. He makes an experimental simulation using n laplace-coins in a one-period pure exchange economy with one security traded and shows that the information level affects the gain and losses on the transaction. For a less informed trader it is rational to switch from an active to a passive investment strategy, and in a market that is less than perfectly efficient it pays to be very well informed.

Importance and the Uniqueness of this Study

The current study is important because to date not much research has been done on this issue. In this study, we take directly into our model the information cost and we assume that traders are risk-neutral. In our premise, the market is somewhat inefficient and there must be a reason for traders to spend a large amount of money on information. Information must have an economic value, and it must be worth being better informed traders. The result possibly solves the equity premium puzzle and enhances asset-pricing in the asymmetric information.

Research Method and Modeling

The previous part presented the literature review as well as the empirical research related to the heterogeneous expectations and asset price. This part will analyze the mathematical model. Let us begin with the assumption, followed by the solution of equilibrium price and finalized with implication and discussion. This research takes a similar approach as in Kyle (1985); the difference is that we extend the Kyle model into two periods with an informed risk-neutral trader and uninformed traders. We will make a two-period model in the secondary market and try to find out the impact of asymmetric information on the asset prices. The informed trader has private information about expected realization value of risky asset and has to trade at a price which does not yet reflect the information.

The Optimum Demand and Equilibrium Price

Model setup

Suppose there is a risky asset in the market and there is a risk-neutral informed trader who has information about the expected realization value of a risky asset at the end of the first and second periods. There is a risk-neutral uninformed trader who has to trade for liquidity reasons or consumption shock. Let us assume that the time span is very short. Therefore, the expected realization value of risky asset in both periods is the same and there is no discounting problem. The informed trader demands a risky asset on the basis of private information that is not known to the other trader when the trade takes place.

The total order flow in the first period consists of the informed trader's order and the uninformed trader's order. The informed trader's strategy is to maximize profit given his private information. The total order flow in the second period comes from the informed trader and the uninformed trader as well. The crucial assumption is that the uninformed trader must not to lose money per se, and is restricted in his ability to buy the market portfolio and has no access to the financial market. Otherwise, it makes possible for him to earn at least as much as risk free rate (Kyle 1985). If he has access to the financial market, he is able to use his security as collateral for a loan at a risk-free rate. This assumption is debatable since large institutional companies are able to get access to the financial market at very low interest rates.[4]

The informed trader is not allowed to submit market order which is condition on the demand of the uninformed trader; nor is he allowed to submit a strategic untruthful order to the market maker. The informed trader will sell an asset and invest in another only if he has private information that the former asset is relatively overvalued at current market prices. This is a little bit different from the argument that prices are not fully revealing. As in Kyle's (1985) model, the demand for the risky asset rises, because the informed trader received good news or because there are large liquidity demands. The informed trader submits a market order given his private information to maximize the expected profit. The risk-neutral informed trader will purchase any amount of risky assets when the asset price equals the expectation of its fundamental value.

The two-period model is an extension of Kyle's (1985) model, where trading takes place in the first and second periods. The informed and uninformed traders submit market orders to the market makers in both periods as follows:

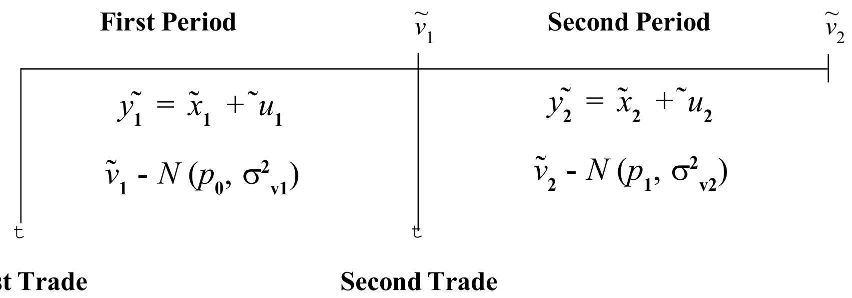
$$\tilde{y}_1 = \tilde{x}_1 + \tilde{u}_1 \qquad (1)$$

$$\tilde{y}_2 = \tilde{x}_2 + \tilde{u}_2 \qquad (2)$$

The informed trader has private information regarding the expected realization value of a risky asset in the two periods. Since the time span is assumed to be very short, the expected realization value would be the same, $\tilde{v}_1 = \tilde{v}_2$. The demand of the uninformed

[4] Many thanks to Klaus Schredelseker and Matthias Bank for pointing out this plausible argument and for giving us a nice clue.

Figure 1. **The Trading Sequence**



trader in the first and the second periods is $\tilde{u}_1, \tilde{u}_2 \sim N(0, \sigma_u^2)$. We assume that the uninformed trader's order is independent of all other random variables, and hence nothing can be learned about the liquidation value of the asset from the uninformed trader's orders. The trading sequence can be seen in the Figure 1.

Let us assume that the market makers determine the price at which they trade the quantity necessary to clear the market. As a competitive profession, the expected profit of market makers is zero. The asset price functions for both periods as standard (Kyle 1985) model is the linear function of the total order flows as follows:

$$P_1 = p_0 + \lambda_1 \tilde{y}_1 \qquad (3)$$

$$P_2 = p_1 + \lambda_2 \tilde{y}_2 \qquad (4)$$

Lambda measures how the market maker learns about the liquidation value of a risky asset from order flows. The market depth is measured by the reciprocal of λ_1 and λ_2 for the first and the second period, respectively. The depth refers to the ability of the market to absorb quantity without having a large impact on price. If a lambda is small, the market is said to be very liquid. It means that an increase in traders' demand has only small impact on the price. The profit maximization strategy of informed traders could be found using the backward induction approach, because a Bayesian Nash equilibrium requires equilibrium strategies to be optimal for each trader given the rational belief. Thus, the informed trader will maximize the value of his private information in both periods.

Lemma 1: Within the structure above, the optimum demand of an informed trader in the first and the second period is as follows:

$$x_1 = \frac{(\lambda_1 - 2\lambda_2)}{\lambda_1^2 - 4\lambda_1\lambda_2}(v_1 - p_0)$$

and

$$x_2 = \frac{v_2 - p_1}{2\lambda_2}$$

Proof: Using the backward induction approach,[5] the optimum quantity order of an informed trader in the first period can be solved as follows:

$$\begin{aligned} &Max_{x_2}\ E\{x_2(v_2 - P_2)\} \\ &= Max_{x_2}\ E\{x_2(v_2 - p_1 - \lambda_2(x_2 + u_2))\} \\ &= Max_{x_2} E\{x_2 v_2 - x_2 p_1 - \lambda_2 x_2^2 - \lambda_2 x_2 u_2\} \end{aligned} \tag{5}$$

We know that $E\ [u_2] = 0$; therefore, we can solve for x_2 by taking the first derivative in respect of x_2 and equating it to zero. This gives the following equation:

$$\begin{aligned} &2\lambda_2 x_2 = v_2 - p_1 \\ &x_2 = \frac{v_2 - p_1}{2\lambda_2} \end{aligned} \tag{6}$$

Now let us substitute Equation (6), the optimum order of the informed trader in the second period, into Equation (5) to find the optimum profit, as:

$$\begin{aligned} &Max_{x_2} E\ \{x_2\ (v_2 - p_1 - \lambda_2\ (X_2 + u_2))\} \\ &= \frac{v_2 - p_1}{2\lambda_2}\left[v_1 - p_1\ \lambda_2\left(\frac{v_2 - p_1}{2\lambda_2}\right)\right] \\ &= \frac{v_2 - p_1}{2\lambda_2}\left[\frac{1}{2}\left(v_1 - p_1\right)\right] \\ &= \frac{1}{4\lambda_2}(v_2 - p_1)^2 \end{aligned} \tag{7}$$

We use the same backward approach to find the optimum demand of the informed trader in the first period. The informed trader acts strategically to optimize his superior information. Now let us substitute the optimum demand of the second period into the first period, and we know that the informed trader faces the same problem of maximizing his expected profit as follows:

$$\begin{aligned} &Max_{x_1}\ E\{x_1(v_1 - P_1) + x_2(v_2 - P_2)\} \\ &= Max_{x1}\ E\Big\{x_1\ (v_1 - p_0 - \lambda_1\ (x_1 + u_1)) \\ &\quad + \frac{1}{4\lambda_2}(v_2 - p_0 - \lambda_1\ (x_1 + u_1))^2\Big\} \\ &= Max_{x1}\ E\Big\{x_1\ (v_1 - p_0 - \lambda_1 x_1 - \lambda_1 u_1) \\ &\quad + \frac{1}{4\lambda_2}(v_2 - p_0 - v_2 - \lambda_1 x_1 - \lambda_1 u_1)_2\Big\} \end{aligned} \tag{8}$$

---

[5] Hirth (1999) provide an excellence example of how to use backward induction to find the optimum demand in a two-period model. In line with the Kyle (1985) approach, he focused on the insider trading and its effect on market liquidity and information efficiency. He proved that when the probability of an earlier publication date increases, the insider trades more and the price reflects more information, but the market liquidity decreases.

Let us simplify by factoring the second term of Equation (8) as $m = v_2 - p_0 - \lambda_1 x_1$ and $n = \lambda_1 u_1$. So the second term is equivalent to $r = (m - n)^2 = m^2 + n^2 - 2mn$. Keep in mind that $E[u_1] = 0$; if we take the first derivative in respect of x_1 and equating it to zero, we are able to solve for x_1 as follows:

$$= Max_{x_1} E\left\{\begin{array}{l} x_1(v_1 - p_0 - \lambda_1 x_1 - \lambda_1 u_1) \\ + \frac{1}{4\lambda_2}(v_2 - p_0 - \lambda_1 x_1)^2 + \frac{1}{4\lambda_2}(\lambda_1 u_1)^2 \\ - \frac{2}{4\lambda_2}(v_2 - p_0 - \lambda_1 x_1)(\lambda_1 u_1) \end{array}\right\}$$

$$= Max_{x_1}\left\{\begin{array}{l} x_1(v_1 - p_0 - \lambda_1 x_1) + \frac{1}{4\lambda_2}(v_2 - p_0 - \lambda_1 x_1)^2 \\ + \frac{\lambda_1^2}{4\lambda_2}\sigma_{u1}^2 \end{array}\right\}$$

(9)

Let us assume the second term of Equation (9) as $K = v_2 - p_0 - \lambda_1 x_1$ thus, $\frac{dK}{dx_1} = -\lambda_1$. We may also simplify the second term as $L = K^2$ and $\frac{dL}{dK} = 2K$. So, the first derivation of the second term is

$$\left(\frac{dK}{dx_1}\right)\left(\frac{dL}{dK}\right) = \frac{dL}{dx_1} = -2\lambda_1(v_2 - p_0 - \lambda_1 x_1)$$

Now take the first derivation of Equation (9) as a whole with respect of x_1, and equating it to zero as:

$$v_1 - p_0 - 2\lambda_1 x_1 - \frac{\lambda_1}{2\lambda_2}(v_2 - p_0 - \lambda_1 x_1) = 0 \qquad (10)$$

As we assume that $v_1 = v_2$, thus we can rearrange and simplify as follows:

$$\frac{\lambda_1^2}{2\lambda_2}x_1 - \frac{4\lambda_1\lambda_2}{2\lambda_2}x_1 =$$

$$\frac{\lambda_1(v_1 - p_0)}{2\lambda_2} - \frac{2\lambda_2(v_1 - p_0)}{2\lambda_2}$$

$$x_1 = \frac{(\lambda_1 - 2\lambda_2)}{\lambda_1^2 - 4\lambda_1\lambda_2}(v_1 - p_0) \qquad (11)$$

Equation (11) shows that the optimum demand in the first period is a linear function of private information in the sense of the difference between the realization value of the risky asset and the current price. It is obvious that when the informed trader knows the realization value would be higher than the current price, $v_1 > p_0$, he will demand the risky asset. The higher the difference between the realization value and the current price, the greater the demand of the informed trader would be. The informed trader not only takes advantage of his information, but also considers the market maker's pricing strategy in both periods as measured by lambda.

Market makers' pricing strategy

The market maker receives the market order from the informed and the uninformed trader and sets up the price to clear the market. The market maker knows the total order but not each order from the trader. As a competitive profession, the market maker is protected from making a positive profit. The market maker will adjust the price whenever the order flow changes, and we assume that the price is a linear function of order flows.

Proposition 1: Under the assumption as set up above, a perfect Bayesian Nash equilibrium in which the market maker's pricing strategy in the first and second period is as follows:

$$\lambda_1 = \frac{(\lambda_1 - 2\lambda_2)(\lambda_1^2 - 4\lambda_1\lambda_2)\sigma^2}{(\lambda_1 - 2\lambda_2)^2\sigma^2 + (\lambda_1^2 - 4\lambda_1\lambda_2)^2\sigma^2_{u1}}$$

and $\lambda_2 = \dfrac{\sigma_{v2}}{2\sigma_{u2}}$

Proof: The next step is to define λ_1 and λ_2 as a measure of how the market maker learns the realization value from the order flows. The reciprocal of lambda measures the market depth; the smaller lambda is, the deeper the market would be. Remember that the market maker set the price to clear the market as a linear function of total order flow. From Equation (3) we know the price function as $P_1 = p_0 + \lambda_1\tilde{y}_1$; this is equal to $P_1 - p_0 = \lambda_1 y_1^{\sim}$ and $P_1 = E\ [\tilde{v}_1 \neq \tilde{y}_1]$.

So, $P_1 - p_0 = E[\tilde{v}_1 - p_0 \neq \tilde{y}_1]$ solving for λ_1 as follows,

$$P_1 - p_0 = \lambda_1 y_1 = E\ [v_1 - p_0 \mid y_1] \tag{12}$$

$$E[\tilde{v}_1 - p_0 | \tilde{y}_1] = \frac{Cov(\tilde{v}_1 - p_0, \tilde{y}_1)}{Var(\tilde{y}_1)}\tilde{y}_1$$

$$\lambda_1 = \frac{Cov(\tilde{v}_1 - p_0, \tilde{y}_1)}{Var(\tilde{y}_1)} \tag{13}$$

We know that $\tilde{y}_1 = x_1 + \tilde{u}_1$, and remember that $v_1 = v_2$; therefore, if we substitute Equation (11) into the demand function, we have the following expression.

$$\tilde{y}_1 = \frac{(\lambda_1 - 2\lambda_2)(\tilde{v}_1 - p_0)}{\lambda_1^2 - 4\lambda_1\lambda_2} + \tilde{u}_1 \tag{14}$$

The market maker does not know the realization value of the risky asset v_1, he only observes the distribution of the expected realization value v_1, thus:

$$\tilde{y}_1 = \left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2}\right)\tilde{v}_1 - \left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2}\right)p_0 + 1\tilde{u}_1$$

$$Var(\tilde{y}_1) = \left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_{21}}\right)^2 \sigma_{v1}^2 + \sigma_{u1}^2$$

and

$$Cov(\tilde{v}_1 - p_0, \tilde{y}_1) = \left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2}\right)\sigma_{v1}^2$$

$$\lambda_1 = \frac{\left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2}\right)\sigma_{v1}^2}{\left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2}\right)^2\sigma_{v1}^2 + \sigma_{u1}^2}$$

$$= \frac{(\lambda_1 - 2\lambda_2)(\lambda_1^2 - 4\lambda_1\lambda_2)\sigma_{v1}^2}{(\lambda_1 - 2\lambda_2)^2\sigma_{v1}^2 + (\lambda_1^2 - 4\lambda_1\lambda_2)^2\sigma_{u1}^2} \quad (15)$$

Following Equation (15) a similar approach could be used to find λ_2 as we know the price is a function of total order flows. Thus, in the second period, the function of the asset price is $P_2 = p_1 + \lambda_2\tilde{y}_2$; so $P_1 - p_1 = \lambda_2\tilde{y}_2$ and $P_1 = E[\tilde{v}_2 | \tilde{y}_2]$. The similar way follows,

$$P_2 - p_1 = E[(\tilde{v}_2 - p_1)|\tilde{y}_2]$$

$$\lambda_2\tilde{y}_2 = E[(\tilde{v}_2 - p_1)|\tilde{y}_2] =$$

$$\frac{Cov(\tilde{v}_2 - p_1, \tilde{y}_2)}{Var(\tilde{y}_2)}\tilde{y}_2 \quad (16)$$

Substitute Equation (6) into the total order flows $\tilde{y}_2 = x_2 + \tilde{u}_2$ and solve for variance and covariance of the total order flows. To find λ_2 we can use a similar approach where the market maker only observes the distribution of the expected realization value of risky asset $\tilde{v}_2$, but does not know the realization value in the second period. Therefore, lambda in the second period can easily be found as follows:

$$\tilde{y}_2 = \frac{\tilde{v}_2 - p_1}{2\lambda_2} + \tilde{u}_2 = \frac{1}{2\lambda_2}(\tilde{v}_2 - p_1) + \tilde{u}_2$$

$$Var(\tilde{y}_2) = \frac{1}{4\lambda_2^2}\sigma_{v2}^2 + \sigma_{u2}^2$$

and

$$Cov(\tilde{v}_2 - p_1, \tilde{y}_2) = \frac{1}{2\lambda_2}\sigma_{v2}^2 \quad (17)$$

$$\lambda_2 = \frac{\frac{2\lambda_2\sigma_{v2}^2}{4\lambda_2^2}}{\frac{\sigma_{v2}^2 + 4\lambda_2^2\sigma_{u2}^2}{4\lambda_2^2}}$$

$$= \frac{2\lambda_2\sigma_{v2}^2}{\sigma_{v2}^2 + 4\lambda_2^2\sigma_{u2}^2} = \frac{\sigma_{v2}}{2\sigma_{u2}} \quad (18)$$

We found that the market depth in the second periods is similar to the Kyle' (1985) one-period model. However, we found that the depth in the first period is not straight forward, because lambda in the first period is a function of the lambda in the second periods and the first period as well. In the second periods, the depth is increasing function of the volatility of the uninformed trader and decreasing in the volatility of the risky asset. The market depth is measured by the reciprocal of lambda. It measures how the demand changes affect the price. The bigger the lambda is, the smaller the market depth would be. Thus, when the volatility of the uninformed trader's demand increases, the informed trader is better-off because he is able to disguise his private information from the price.

Equilibrium price under costly information

Up to this point, the information cost is not yet considered. Let us assume that the informed trader spends as much as c to acquire the information. Being better informed, his expected maximum profit presumably will increase and he will stop collecting the information until the marginal cost equals the marginal benefit of being informed. Assuming that the time span is very short, the interest rate between first and second period is zero. Therefore, the expected realization value of risky asset is the same in both periods, $\tilde{v}_1 = \tilde{v}_2$. The informed trader will maximize his private information and use a similar approach, while the $E\,[u_2]= 0$, we can find the solution as follows:

$$\begin{aligned} &\underset{x_2}{Max}\; E\{(x_2(v_2 - p_1 - \lambda_2(x_2 + u_2)) - c\} \\ &= \underset{x_2}{Max}\; E\{(x_2(v_2 - p_1 - \lambda_2 x_2^2 - \lambda_2 u_2) - c\} \\ &= \underset{x_2}{Max} E\{(x_2 v_2 - p_1 x_2 - \lambda_2 x_2^2 - \lambda_2 u_2 x_2) - c\} \end{aligned} \tag{19}$$

Taking the first derivation in respect of x_2 and equating it to zero, we are able to find the optimum order of the informed trader in the second period as follows:

$$v_2 - p_1 - 2\lambda_2 x_2 = 0$$

$$\text{and } x_2 = \frac{v_2 - p_1}{2\lambda_2} \tag{20}$$

The optimum order of the informed trader in the second period is not affected by the information cost. The information cost, c then disappears from the first derivation in respect of x_2 unless the information cost is a function of the number of shares or investment size. The argument is not quite logical if the information cost is a constant, regardless of the number of shares traded. The more plausible argument is that the larger the investment, the more likely traders will spend money on information to protect their investment. Therefore, the information cost must be a function of the number of shares traded or investment size. If it is true, then the information cost, c from Equation (19) must be within the bracket.

Lemma 2: Under costly information acquisition as structured above, the optimum demand of the informed trader in the first and the second period is as follows:

$$x_1 = \frac{(\lambda_1 - 2\lambda_2)(v_1 - p_0) - \lambda_1(c)}{\lambda_1^2 - 4\lambda_1\lambda_2}$$

$$\text{and } x_2 = \frac{v_2 - p_1 - c}{2\lambda_2}$$

Proof: Now let us modify Equation (19) by assuming the information cost is a linear function of the investment in the sense of the demand for risky asset. It is true that the informed trader has to bear the information cost before submitting the order. But it is plausible to assume that a trader who has a large investment is willing to spend more for information than other traders who have a smaller investment. Using a similar approach we can find the optimum demand of the informed trader in the second period as follows:

$$x_2 = \frac{v_2 - p_1 - c}{2\lambda_2} \qquad (21)$$

The more detailed proof is omitted due to the very simple case and it can be solved by modifying the previous proof. The optimum order of the informed trader is a linear function of the information accuracy minus the information cost. The more accurate the information is, the larger the order of the informed trader will be. The informed trader will continue to collect the information as long as the information value is higher than the cost, $(v_2 - p_1) > c$. The optimum profit in the second period given optimum orders, x_2 is as follows:

$$\text{Profit} = \left\{ \frac{v_2 - p_1 - c}{2\lambda_2}\left(v_2 - p_1 - \lambda_2\left(\frac{v_2 - p_1 - c}{2\lambda_2} + u_2\right) - c\right)\right\}$$

$$= \frac{1}{4\lambda_2}(v_2 - p_1 - c)^2 \qquad (22)$$

Let us substitute Equation (22) into the optimization function in the first period; we know that the in the first period, the informed trader faces a similar profit maximization problem. As we assume that $E[\tilde{u}_1, \tilde{u}_2] = 0$ and we are able to rearrange Equation (23) and solve for x_1, the optimum order of the informed trader in the first period is as follows:

$$Max_{x_1} E\{x_1(v_1 - P_1) + x_2(v_2 - P_2)\}$$

$$= Max_{x_1} E\left\{\begin{aligned} & x_1(v_1 - p_0 - \lambda_1 x_1 - \lambda_1 \tilde{u}_1) \\ & + \frac{1}{4\lambda_2}(v_1 - p_0 - \lambda_1 x_1 - \lambda_1 \tilde{u}_1 - c)^2 \end{aligned}\right\}$$

$$= Max_{x_1}\left\{\begin{aligned} & v_1 x_1 - p_0 x_1 - \lambda_1 x_1^2 \\ & + \frac{1}{4\lambda_2}(v_1 - p_0 - \lambda_1 x_1 - c)^2 + \frac{\lambda_1^2}{4\lambda_2}\sigma_u^2 \end{aligned}\right\} \qquad (23)$$

We can use the similar proof as in Lemma 1 to find the optimum demand in the first period. Thus, some steps were omitted.

$$v_1 - p_0 - 2\lambda_1 x_1 - \frac{\lambda_1}{2\lambda_2}(v_1 - p_0 - \lambda_1 x_1 - c) = 0$$

$$\left(\frac{\lambda_1^2 - 4\lambda_1\lambda_2}{2\lambda_2}\right)x_1 = \left(\frac{\lambda_1 - 2\lambda_2}{2\lambda_2}\right)(v_1 - p_0) - \frac{\lambda_1}{2\lambda_2}(c)$$

$$x_1 = \frac{(\lambda_1 - 2\lambda_2)(v_1 - p_0) - \lambda_1(c)}{\lambda_2^2 - 4\lambda_1\lambda_2} \qquad (24)$$

Equations (21) and (24) show that the optimum demands of the informed trader decreases in the information cost. The market makers' pricing strategy has an impact on the optimum demand as well. It is obvious that if the realization value is equal to the current price $v_1 = p_0$, no one will be willing to collect the private information.

Proposition 2: Given the optimum demand in the first and the second period, the market maker set the pricing strategy as follows:

$$\lambda_1 = \frac{(\lambda_1 - 2\lambda_2)(\lambda_1^2 - 4\lambda_1\lambda_2)\sigma^2_{v1}}{(\lambda_1 - 2\lambda_2^2)^2\sigma^2_{v1} + (\lambda_1^2 - 4\lambda_1\lambda_2)^2\sigma^2_{u1}}$$

and $\lambda_2 = \dfrac{\sigma^2_{v1}}{2\sigma^2_{u1}}$

Proof: We can use a similar approach to prove the above proposition. Equations (21) and (24) can be used to find λ_1 and λ_2 as well by substituting into the demand function, as we know that the price is a linear function of total order flows. Thus, in the second period the price of the risky asset is $P_2 = p_1 + \lambda_2 y_2$. So, $P_2 - p_1 = p_1 + \lambda_2\tilde{y}_2$, and $P_1 = E[\tilde{v}_2 | \tilde{y}_2]$. The similar way follows,

$$P_2 - p_1 = E[(v_2 - p_1) | y_2].$$

$$\lambda_2\tilde{y}_2 = E[(v_2 - p_1) | y_2] = \frac{Cov(\tilde{v}_2 - p_1, \tilde{y}_2)}{Var(\tilde{y}_2)} y_2 \quad (25)$$

If we substitute Equation (21) into the total order flow $\tilde{y}_2 = x_2 + \tilde{u}_2$ and remember that the market maker only observe the distribution of the realization value of risky asset $\tilde{v}_2$, we can solve for lambda giving the following result:

$$\tilde{y}_2 = \frac{\tilde{v}_2 - p_1 - c}{2\lambda_2} + \tilde{u}_2$$

$$\tilde{y}_2 = \frac{1}{2\lambda_2}(\tilde{v}_2 - p_1) - \frac{1}{2\lambda_2}c + \tilde{u}_2 \quad (26)$$

$$Var(\tilde{y}_2) = \frac{1}{4\lambda_2^2}\sigma^2_{v2} + \sigma^2_{u2}$$ and

$$Cov(\tilde{v}_2 - p_1, \tilde{y}_2) = \frac{1}{2\lambda_2}\sigma^2_{v2} \quad (27)$$

$$\lambda_2 = \frac{2\lambda_2\sigma^2_{v2}}{\sigma^2_{v2} + 4\lambda_2^2\sigma^2_{u2}} = \frac{\sigma_{v2}}{2\sigma_{u2}} \quad (28)$$

We know that as in the Kyle (1985) model $1/2\lambda = \beta$ and beta measures the aggressiveness of the informed trader. In the second period, the market depth –as measured by the reciprocal of lambda– is a linear function of the volatility of the uninformed trader and decreases in the volatility of the risky asset. The more volatile the uninformed trader in the market is, the more aggressive the informed trader would be. As we assume that the market maker sets up the price as a linear function of order flows; so, when the volatility of uninformed trader increases, the price become even less efficient and the depth increases. A similar approach can be applied to find lambda in the first period.

Let us substitute the demand of the informed trader and the uninformed trader in the first period and solve for the variance of total order flows and covariance between the expected realization value of the risky asset in the first period and the total order flows. Then, we are able to find lambda as follows:

$$\tilde{y}_1 = \frac{(\lambda_1 - 2\lambda_2)(\tilde{v}_1 - p_0) - \lambda_1(c)}{\lambda_1^2 - 4\lambda_1\lambda_2} + \tilde{u}_1$$

$$Var(\tilde{y}_1) = \left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2}\right)^2 \sigma_{v1}^2 + \sigma_{u1}^2$$

and

$$Cov(\tilde{v}_1 - p_1, \tilde{y}_1) = \left(\frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2}\right)\sigma_{v1}^2$$

$$\lambda_1 = \frac{(\lambda_1 - 2\lambda_2)(\lambda_1^2 - 4\lambda_1\lambda_2)\,\sigma_{v1}^2}{(\lambda_1 - 2\lambda_2^2)^2\sigma_{v1}^2 + (\lambda_1^2 - 4\lambda_1\lambda_2)^2\sigma_{u1}^2} \tag{29}$$

From Equation (29) we find that the market depth in the first period is a linear function of the variance of uninformed traders' demand and inversely related in the variance of the asset value. However, we are not able to find the closed form solution for lambda in the first period because the lambda in the first period is a function of the lambda in the first and the second periods. This problem is open for a further research. As in Kyle (1985), lambda represents how the market maker learns about the liquidation value of the risky asset from the order flows. The reciprocal of lambda coefficient is interpreted as the market depth, which refers to the ability of the market to absorb quantity without having a large effect on price. If the lambda coefficient is small, then the market is very liquid. This means an increase in the trader's demand has only a small impact on the price.

The Implication and Discussion

Based on the Kyle models, we find that the market depth in the second period as measured by the reciprocal of lambda is a linear function of the uninformed traders and inversely related to the private information. The higher the information level, the better the estimate of asset value and the smaller the error would be. The market depth increases whenever the informed trader has better information in the sense that the informed trader knows the price does not yet reveal the expected realization value of risky asset. The informed trader has to trade immediately whenever he has private information that is not yet reflected in the price. Otherwise, his information become valueless.

From the uninformed traders' point of view, when the demand of the informed trader rises, the price risky asset becomes more efficient. It is because the informed trader has private information and has to take advantage of his information. The uninformed trader then able to infer the information better from the price and he is even more willing to trade. The increase in uninformed trader's demand provide an opportunity for the informed trader to camouflage the trade and hide his private information. However, we have a little bit difficulty to draw direct conclusion for the price behavior and the market depth in the first period since we do not have a close-simple form solution. We find that the market depth in the first period is affected by the market depth in the second period. This problem remains open for further research.

The Lemma 2, shows that the costly information acquisition has an impact on the optimum demand of the informed trader as long as the information cost is a linear function of investment size. But when we assume that the information cost is fixed, no matter the investment size is, the information cost does not affect the equilibrium demand. The question then, is it plausible to assume that the information cost is a linear function of investment size? We believe that the larger the investment is, the bigger the risk faced by trader would be. Therefore, trader willing to bear higher information cost to minimize the risk. The information cost can be defined in terms of money, time and effort to acquire and to analyze the information. This argument also supports the empirical evidence that institutional investors are willing to hire experts and analysts just to make any better valuation of the risky asset.

The optimum demand in the first period shows that the informed trader takes into account the pricing strategy in both periods as measured by lambda. It is plausible since the price of the risky asset in the second period, $P_2 = p_1 + \lambda_2 (x + \tilde{u})$ is a function of the price in the first period. The price in the first period becomes public knowledge and the price in the second period must reflect the private information of the informed trader in the second period. Lambda also indicates how the market maker adjusts the price to the changes of order flow. It is true that indirectly we are assume that the informed trader makes profit at the expense of uninformed trader. This argument may raise some doubt. However, we believe that in the market not all traders have access to the information and some of them do trade simply because they like it to do so or due to other reasons. Our model disserved to be tested empirically and open for further research.

Conclusion

In this research we model trading behavior and examine the impact of heterogeneous expectations on asset prices. We explore a market when informed traders have private information about the expected realization value of a risky asset. We extend Kyle's (1985) one-period model to a two-period model. The optimum demand in the second period is similar to what have been shown in Kyle. In the first period, the optimum demand of the informed trader is a linear function of the private information as measured by the difference between the realization value of a risky asset and the current price, v_1 - p_0. The informed trader takes into account not only the pricing function in the first period, but also the pricing function in the second period.

The price of risky asset is an increasing function of the volatility of the expected realization value, and it decreases in the volatility of the uninformed traders'demand. When we assume that the information cost is a function of investment size, we find that the costly information acquisition has an impact on the optimum demand of the informed trader. The information cost has indirect impact on the price behavior through the optimum demand of informed trader. It can be seen from lambda for both periods, that the information cost did not appear on the equations. In other words, market makers do not consider the information cost directly in setting the price. However, if we assume that the information cost is independent of the investment size, we do not find that the information cost does have an impact on the demand of the risky asset.

References

Admati, R. A. 1985. A noisy rational expectation equilibrium for multi asset securities markets. *Econometrica* 53: 629-657.

Admati, R. A., and P. Pfleiderer. 1987. The value of information in speculative trading. *Research paper.* Stanford University

Admati, R. A., and P. Pfleiderer. 1988. A theory of intraday patterns: Volume and price variability. *Review of Financial Studies* 1: 3-40.

Bagnoli, M., S. Viswanathan, and C. Holden. 2001. On the existence of linear equilibria in models of market making. *Mathematical Finance* 11: 1-31.

Ball, R., and S.P. Kothari. 1989. Nonstationary expected return: Implication for test of market efficiency and serial correlation in returns. *Journal of Financial Economics* 25: 51-74.

Banz, R. 1981. The relationship between return and market value of common stock. *Journal of Financial Economics*, 9: p.3-18.

Barlow, R.J. 1989. *Statistic: A Guide to the Use of Statistical Methods in the Physical Sciences*. Singapore: John Wiley and Sons.

Barry B. C., and S. J. Brown. 1984. Differential information and the small firm effect., *Journal of Financial Economics* 14: 283-294.

Bassembinder, H., K. Chan, and P. J. Seguin. 1996. An empirical examination of information, differences of opinion and trading activity. *Journal of Financial Economics* 40: 105-134.

Basu, D. 2001. Conditioned diffusions and the equity premium. *Working Paper*. University of Warwick.

Basu, S. 1977. Investment performance of common stock in relation to their price earning ratios: A test of market efficiency. *Journal of Finance* 32: 663-682.

_____________. 1983. The relationship between earnings' yield, market value, and return for NYSE common stocks. *Journal of Financial Economics* 12: 129-156.

Black, F. 1986. Noise. *Journal of Finance* 41: 529-543.

Blume E. L., and D. Easley. 1982. Learning to be rational. *Journal of Economic Theory* 26: 340-351.

_____________. 1990. Implementation of Walrasian expectations equilibria. *Journal of Economic Theory* 51: 207-227.

Blume E. L., D. Easley, and M. O'Hara. 1994. Market statistics and technical analysis: The role of volume. *Journal of Finance* 49: 153-181.

Campbell Y. J., and A. S. Kyle. 1993. Smart money, noise trading and stock price behaviour. *Review of Economic Studies* 60: 1-34.

Cass, D., and J. E. Stiglitz. 1970. The structure of investor preferences and asset returns, and separability in portfolio allocation: A contribution to the pure theory of mutual funds. *Journal of Economic Theory* 2: 122-160.

_____________. 1972. Risk aversion and wealth effects on a portfolio with many assets. *Review of Economic Studies* 39: 331-354.

DeBondt, F. W., and T. Thaler. 1985. Does the stock market overreact? *Journal of Finance* 40: 793-805.

Diamond, W. D., and R. E. Verrecchia. 1987. Constraints on short selling and asset price adjustment to private information. *Journal of Financial Economics* 18: 227-311.

_____________. 1981. Information aggregation in a noisy rational expectation economy. *Journal of Financial Economics* 9: 221-235.

Dow, J., and G. Garton. 1995. Profitable informed trading in a simple general equilibrium model of asset pricing. *Journal of Economic Literature* 33: 327-339.

Easley, D., N. M. Kiefer, and M. O'Hara. 1997. The information content of the trading process. *Journal of Empirical Finance* 4: 159-186.

Easley, D., and M. O'Hara. 2001. Information and cost of capital. *Working Paper*. Cornell University, USA.

Easley, D., S. Hvidkjare, and M. O'Hara. 2002. Is information risk a determinant of asset return? *Journal of Finance* 57: 2185-2221.

Fama, F. E. 1970. Efficient capital market: A review of theory and empirical work. *Journal of Finance* 25: 383-417.

___________. 1991), "Efficient Capital Market: II," *Journal of Finance*, Vol.46, p.1575-1617.

Fama, F. Eugene and Kenneth R. French (1992), "The Cross Section of Expected Stock Returns," *Journal of Finance*, Vol.46, p.427-466.

_____________. 2002. The equity premium. *Journal of Finance* 57: 637-660.

Figlewski, S. 1982. Information diversity and market behavior. *Journal of Finance* 37: 87-102.

Foster, F. D., and S. Viswanathan. 1993. The effect of public information and competition on trading volume and price volatility. *Review of Financial Studies* 6: 23-56.

French, R. K. 1980. Stock returns and weekend effect. *Journal of Financial Economics* 8: 55-69.

Glosten, L., and P. Milgrom. 1985. Bid-ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics* 14: 71-100.

Gorton, B. G., and G. G. Pennacchi. 1993. Security baskets and index-linked securities. *Journal of Business* 66: 1-27.

Grossman, J. S. 1976. On the efficiency of competitive stock markets where traders have diverse information. *Journal of Finance* 31: 573-585.

_____________. 1978. Further results on the informational efficiency of competitive stock markets. *Journal of Economic Theory* 18: 81-101.

Grossman, J. S., and J. E. Stiglitz. 1976. Information and competitive price system. *American Economic Review* 66: 256-253.

_____________. 1980. On the impossibility of informationally efficient markets. *American Economic Review* 70: 393-408.

Hasbrouck, J. 1991. Measuring the information content of stock trades. *Journal of Finance* 46: 179-207.

Hellwig, F. M. 1980. On the aggregation of information in competitive markets. *Journal of Economic Theory* 22: 477-498.

_____________. 1982. Rational expectation equilibrium with conditioning on past prices: A mean-variance example. *Journal of Economic Theory* 26: 279-312.

Hess, J. P., and M. R. Reinganum. 1979. Efficient capital markets. *Handbook of Financial Economics:* 3-12. North Holland Publishing.

Hirth, H. 1999. Insiderhandel bei unsicherem verˆffentlichungszeitpunkt – Insider trading with uncertain date of publication. *Operation Research (OR) Spectrum* 21: 81-96.

Holden, W. C., and A. Subrahmanyam. 1992. Long lived private information and imperfect competition. *Journal of Finance* 47: 247-270.

Jackson, O. M. 1991. Equilibrium, price formation and the value of private information. *Review of Financial Studies* 4: 1-16.

Jackson O, M., and J. Peck. 1997. Asymmetric information in a competitive market game: Reexamining the implications of rational expectations. *Working Paper*. Ohio State University.

Jarrow, R. 1980. Heterogeneous expectations, restrictions on short sales, and equilibrium asset prices. *Journal of Finance* 35: 1105-1113.

Jensen, C. M. 1978. Some anomalous evidence regarding market efficiency. *Journal of Financial Economics* 78: 95-101.

Jegadesh, N., and S. Tirman. 1993. Return to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance* 48: 65-91.

Keim, B. D. 1983. Size-related anomalies and stock return seasonality: Further empirical evidence. *Journal of Financial Economics* 12: 13-32.

Korajczyk, R., D. J. Lucas, and R. L. McDonald. 1991. The effect of information releases on the pricing and timing of equity issues. *Review of Financial Studies* 4: 685-708.

Koski L. J., and R. Michaely. 2000. Prices, liquidity and the information content of trade. *Review of Financial Studies* 13: 659-696.

Kyle, S. A. 1985. Continuous auction and insider trading. *Econometrica* 53:1315-1335.

_____________. 1989. Information speculation with imperfect competition. *Review of Economic Studies* 56: 317-355.

Lakonishok, J., and I. Lee. 2001. Are insider traders informative? *Review of Financial Studies* 14: 79-111.

Lintner, J. 1965. The valuation of risk assets and the selection of risky investment in stock portfolios and capital budgets. *The Review of Economics and Statistics* 47: 13-37.

_____________.1969. The aggregation of investor's diverse judgments and preferences in purely competitive security markets. *Journal of Financial and Quantitative Analysis* 4: 347-400.

M., Harry. 1952. Portfolio selection. *Journal of Finance* 71: 77-91.

_____________. 1952. The utility of wealth. *Journal of Political Economy* 52: 151-58.

_____________. 1959. Portfolio selection: Efficient diversification of investment, a Cowles foundation monograph. Yale University Press.

_____________. 1991. Foundations of portfolio theory. *Journal of Finance* 46: 469-477.

Mehra, R., and E. C. Prescott. 1985. The equity premium: A puzzle. *Journal of Monetary Economics* 15: 145-161.

Michaely, R., R. Thaler, and K. Womack. 1995. Price reactions to dividend initiations and omissions: Overreaction or drift? *Journal of Finance* 50: 573-608.

Milgrom, R. P. 1981. Rational expectation, information acquisition and competitive bidding. *Econometrica* 49: 921-943.

Milgrom, R. P., and N. Stokey. 1982. Information, trade and common knowledge. *Journal of Economic Theory* 26: 17-27.

Miller, M. E. 1977. Risk, uncertainty, and divergence of opinion. *Journal of Finance* 32: 1151-1169.

N˘ldeke, G., and Th. Tr˘ger. 2001. Existence of linear equilibria in the Kyle model with multiple informed traders. *Discussion Paper*. Bonn: Bonn Graduate School of Economics, University of Bonn.

O"Hara, M. 1995. Market micro structure theory. Massachusetts, USA: Blackwell Publisher.

Philippatos, C. G. 1979. Mean-variance portfolio selection strategies. *Handbook of Financial Economics*: *309-386.* North Holland Publishing.

Pindyck, S. R., and D. L. Rubinfeld. 1991. Econometric models and economic forecast. New York: MCGraw-Hill Publisher.

Reinganum, R. M. 1983. The anomalous stock market behavior of small firms in January. *Journal of Financial Economics* 12: 89-104.

Roll, R. 1977. A critic of the asset pricing theory's tests: On the past and potential testability of the theory. *Journal of Financial Economics* 4; 129-176.

Ross. S. A. 1989. Information and volatility: The no-arbitrage martingale approach to timing and resolution irrelevancy/ *Journal of Finance* 44: 1-17.

Rozeff, M., and W.R. Kinney. 1976. Capital market seasonality: The case of stock returns. *Journal of Financial Economics* 3: 379-402.

Rubinstein, M. 1974. An aggregation theorem for securities markets. *Journal of Financial Economics* 1: 225-244.

______________. 2000. Rational markets: Yes or no? The affirmative case. *Working Paper*. Berkeley: University of California.

Schredelseker, K. 1980. Unequally distributed information and stock market theory. *Wuppartel* 50: 1-19.

______________. 1982. Towards an equilibrium degree of market efficiency. *Working Paper*: 1043-1061. Innsbruck: University of Innsbruck.

______________. 1997. On the value of information in financial decisions: A simulation approach. *Working Paper*. Innsbruck: University of Innsbruck.

Seyhun, N. 1986. Insider Profit, cost of trading, and market efficiency. *Journal of Financial Economics* 16: 189-212.

Sharpe, F. W. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 19: 425-442.

______________. 1972. Risk, market sensitivity, and diversification. *Financial Analysts Journal* (January/February): 74-79.

Spiegel, M., and A. Subrahmanyam. 1992. Informed speculation and hedging in a noncompetitive securities market. *Review of Financial Studies* 5: 307-329.

Stiglitz, J. E. 1972. On the optimality of the stock market allocation of investment. *Quarterly Journal of Economics* 86: 25-60.

Subrahmanyam, A. 1991. Risk aversion, market liquidity, and price efficiency. *Review of Financial Studies* 4: 417-441.

Varian, R. H. 1989. Difference of opinion in financial markets. *Financial Risk: Theory, Evidence and Implications:* 3-37. Boston: Kluwer Academic Publishers.

Verrecchia, E. R. 1982. Information acquisition in a noisy rational expectation economy. *Econometrica* 50: 1415-1430.

Wang, J. 1993. A Model of intertemporal asset price under asymmetric information. *Review of Economic Studies* 60: 249-282.

Weil, P. 1989. The premium puzzle and the risk free rate puzzle. *Journal of Monetary Economics* 24: 401-421.

Williams, T. J. 1977. Capital asset prices with heterogeneous beliefs. *Journal of Financial Economics* 5: 219-239.