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WHY MATHEMATICS SHAPES REALITY: A PHILOSOPHICAL INQUIRY

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Abstrak

Sebagian besar diskusi dalam filsafat matematika didominasi oleh pertanyaan-pertanyaan terkait sifat entitas dalam matematika, seperti bilangan dan himpunan, sementara hanya sedikit perhatian diberikan untuk matematika terapan. Padahal, matematika telah memainkan peran yang sangat penting dalam perkembangan ilmu pengetahuan alam, menunjukkan bahwa filsafat matematika yang ideal harus dapat menjelaskan keefektifan luar biasa matematika dalam mendeskripsikan dunia nyata. Dua aliran utama filsafat matematika, yaitu Platonisme dan Nominalisme, telah mengabaikan isu tersebut dan tampaknya tidak mampu memberikan penjelasan memadai tentang keberhasilan penerapan matematika dalam ilmu fisik. Akan tetapi, keterbatasan ini tidak bersifat menyeluruh dalam berbagai pendekatan filosofis. Keterbatasan ini secara khusus mencerminkan kelemahan Platonisme dan Nominalisme dalam menghubungkan entitas matematika dengan realitas empiris. Artikel ini menyelidiki filsafat matematika dari sudut pandang alternatif, khususnya pendekatan Antroposentris oleh Steiner dan Realisme Aristoteles oleh Franklin, yang menawarkan kerangka pemahaman yang menjanjikan untuk memahami hubungan mendalam antara matematika dan realitas empiris. Preferensi terhadap pendekatan alternatif ini didasarkan pada potensinya untuk menjelaskan keberhasilan matematika sebagai alat dalam ilmu pengetahuan, dengan menekankan penerapan matematika yang selaras dengan konteks ilmiah. Hasil penelitian menunjukkan bahwa Realisme Aristoteles memberikan kerangka yang lebih kuat dalam menjelaskan keberhasilan matematika sebagai aplikasi empiris *dibandingkan pendekatan lain. Realisme Aristoteles muncul sebagai pendekatan yang unggul dalam filsafat matematika, dengan menempatkan penerapan matematika sebagai inti dari pemahaman filosofisnya.*

Kata kunci: *filsafat matematika, matematika terapan, platonisme, nominalisme, realisme Aristoteles.*

Abstract

Most discussions in the philosophy of mathematics have been dominated by questions concerning the nature of mathematical entities, such as numbers and sets, while comparatively little attention has been given to the applicability of mathematics. Yet mathematics has played an indispensable role in the development of the natural sciences, suggesting that any complete philosophy of mathematics must account for its remarkable effectiveness in describing the physical world. Two major schools of thought, namely Platonism and Nominalism, have largely neglected this issue and seem unable to provide a satisfactory explanation for the tremendous success of mathematics in the physical sciences. However, this limitation does not apply universally across all philosophical approaches. This limitation specifically reflects the weakness of Platonism and Nominalism in connecting mathematical entities to empirical reality. In this article, we investigate the philosophy of mathematics from the standpoint of alternative views, particularly Steiner's Anthropocentric approach and Franklin's Aristotelian Realism, which offer promising frameworks for understanding the deep connections between mathematics and empirical reality. This preference for alternative approaches is justified by their potential to explain the effectiveness of mathematics as a tool in science, emphasizing its applicability and alignment with scientific contexts. The result of this study indicates that Aristotelian Realism provides a more robust framework for explaining the empirical success of mathematics compared to other approaches. Aristotelian Realism stands out as a superior philosophy of mathematics, centering its applicability as the core of its philosophical understanding.

Keywords: *philosophy of mathematics, applied mathematics, platonism, nominalism, Aristotelian realism.*

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INTRODUCTION

What is mathematics really about? Philosophers have debated this question for millennia, yet no final agreement has been reached. For many years, two dominant philosophies, Platonism and Nominalism, have fiercely criticized each other, highlighting each other's flaws. However, amid this controversy, the issue of the applicability of mathematics has often been overlooked (Franklin, 2014a).

The extraordinary effectiveness of mathematics in describing the natural world, as demonstrated by Newton's use of calculus to formulate the laws of motion and gravitation, has been called a "miracle," and there is still no widely accepted explanation for why this is the case (Wigner, 1995). This situation is undesirable. The applicability of mathematics should not be left as an unexplained mystery but must be addressed by any complete philosophy of mathematics. Given the central role mathematics plays in science and technology, a credible account of its application is essential. Philosophies that fail to explain this connection fall short of providing a full understanding of mathematics.

Both Platonism and Nominalism struggle with this challenge. Platonism posits that mathematical objects exist independently in a timeless, non-physical realm (Gödel, 1951). While this view accounts for the objectivity of mathematics, it leaves a troubling gap: how do entities beyond space and time so accurately describe the physical world? Nominalism, on the other hand, denies the independent existence of mathematical objects and treats mathematics as a human invention. Yet it also struggles to explain why human-made conventions should fit the structures of nature so precisely (Bueno, 2013).

As an alternative perspective, meta-mathematics explores the formal properties and consistency of mathematical systems themselves, such as through Gödel's incompleteness theorems, offering a self-referential approach to understanding mathematics beyond its empirical or ontological status. However, this manuscript does not delve into this area in detail, as its focus remains on applied mathematics and its philosophical implications.

The rapid evolution of artificial intelligence (AI) in 2025 has brought mathematics to the forefront of contemporary scientific and technological debates, offering a fresh lens through which we examine its applicability. Recent advancements, such as AI systems assisting in solving complex mathematical problems and proposing new theorems, underscore mathematics' indispensable role as the foundation of AI technologies, including machine learning and neural networks. These systems rely on mathematical principles like linear algebra, calculus, and probability to process data, optimize models, and predict outcomes, yet debates persist about whether AI can truly replicate human mathematical intuition or merely mimic it through computational power. This raises profound philosophical questions: if AI can outperform humans in specific mathematical domains, does this validate Platonism's abstract realism, Nominalism's human-constructed view, or suggest a new paradigm?

As both traditional views appear inadequate when it comes to the applicability problem, a new approach is needed, one that puts the question of application at the center of philosophical inquiry. A philosophy of mathematics must not only account for the existence and nature of mathematical entities but also explain their remarkable usefulness in the empirical sciences, a necessity underscored by Wigner's (1995) observation of the 'unreasonable effectiveness' of mathematics in natural sciences, Steiner's (1998) emphasis on its applicability as a philosophical problem, and Franklin's (2014a) argument for a realist framework that integrates empirical success. This dual focus is supported by the historical success of mathematics in advancing natural sciences and the critiques of traditional views like Platonism and Nominalism for overlooking this aspect (Colyvan, 2001a).

To begin addressing this issue, we first survey several major schools of thought, focusing particularly on Platonism with respect only to mathematics. Since it has been the most influential view, we review various forms of Platonism and select one version for further discussion. Supporting arguments for Platonism, such as the objectivity and necessity of mathematical truth, are considered alongside serious objections, especially the problem of applicability.

Next, we turn to Nominalism, looking at its two main variants: Logicism and Formalism. Logicism, as advanced by Gottlob Frege and Bertrand Russell, seeks to ground mathematics in logic (Tennant, 2023). Frege's Begriffsschrift (1879) introduced a formal system to derive arithmetic from logical axioms, laying the foundation for this approach. This effort was further developed in Russell and Alfred North Whitehead's Principia Mathematica (1910), which aimed to formalize the reduction of mathematics to logical principles through a rigorous symbolic framework. Furthermore, formalism views mathematics as the manipulation of symbols according to rules. While each avoids certain metaphysical commitments, both fail to explain the applicability of mathematics to the physical world, which is our primary concern. For instance, Frege and Russell’s logical foundations, while rigorous, do not account for why mathematical structures align with empirical phenomena.

Recognizing these shortcomings, we then introduce Aristotelian realism as a promising alternative. This view holds that mathematical structures are abstractions from real patterns found within the physical world itself (Franklin, 2014a). Aristotelian realism, still relatively young as a philosophical approach, seems well-positioned to address the applicability problem, as it directly connects mathematics to empirical reality.

Building on this foundation, we discuss different accounts of mathematical applicability. After establishing that Platonism and Nominalism are inadequate, we turn to alternatives that offer more promise. First, we examine Mark Steiner's anthropocentric view. Steiner suggests that the effectiveness of mathematics arises from human reasoning patterns, such as symmetry and analogy, which scientists use to extend known results to new domains. This perspective may seem unclear at first; a more detailed explanation and its implications are explored in the following sections.

We then develop the Aristotelian realist account further, showing how it naturally explains the fit between mathematics and the physical world. In this view, mathematics works because it describes real features of reality, accessible through observation and abstraction.

Finally, we turn to the tools of applied mathematics: models and simulations. We explore different types of models: physical, mathematical, and computational, and discuss the role of abstraction, idealization, and approximation in modeling complex systems. Simulations, in particular, demonstrate how mathematical models bridge theory and empirical data, further emphasizing the need for a philosophy that takes applied mathematics seriously.

The applicability problem highlights a critical gap in traditional philosophies of mathematics. By examining alternative approaches, particularly Aristotelian realism, we move toward a more complete and satisfying understanding of mathematics, one that places its remarkable applicability at the heart of philosophical inquiry.

DISCUSSION

1. Platonism in the Philosophy of Mathematics

Platonism has evolved since Plato first proposed that mathematical entities exist in an abstract, non-spatiotemporal realm. Contemporary mathematicians who identify as Platonists often reject Plato's literal interpretation. Øystein (2024) summarizes Platonism with three theses: mathematical objects exist, they are abstract, and they are mind-independent. Full-blooded Platonism, as Franklin (2014a) describes, posits that universals exist beyond our causal world.

Gödel (1951) represents a strong version of Platonism, asserting that mathematics describes a non-sensual reality. The Continuum Hypothesis, proposed by Cantor in 1878, addresses the cardinality of the continuum (the set of real numbers), asking whether there exists a set with a cardinality strictly between that of the integers ($\aleph_0$) and the continuum ($2^\aleph_0$, also denoted as c). Cantor conjectured that no such set exists, meaning there is no intermediate infinite cardinality, but this remains unproven within the standard Zermelo-Fraenkel set theory with the Axiom of Choice (ZFC). Gödel believes that the Continuum Hypothesis must be either true or false independently of its provability, suggesting that current axioms (e.g., Zermelo-Fraenkel) are insufficient and proposing new axioms to uncover set-theoretic reality.

Alternative perspectives include Davies (2007), who defines Platonism as belief in a mathematical realm outside space and time. However, Gardner (2009) criticizes this, favoring the view that Platonism recognizes mathematical facts as objective but immanent in human experience.

Artstein (2010) proposes "applied Platonism," where mathematical realities approximate physical phenomena, bridging abstract mathematics and empirical science. Although similar to Aristotelian realism, applied Platonism still emphasizes that mathematical statements "stand alone”. Øystein (2024) also discusses "plenitudinous Platonism," which, by the plenitude principle, posits that all possible mathematical objects consistent with any mathematical theory exist in some universe. For example, it holds that there are distinct universes of sets where the Continuum Hypothesis is true and others where it is false, with no single universe being metaphysically privileged. This view is in contrast with traditional Platonism’s assertion of a unique mathematical reality and is proposed to simplify mathematical epistemology by ensuring that consistent theories are true in some universe. This resolves epistemological concerns but diverges from traditional singular-universe Platonism. Despite its variations, there remains no consensus among mathematicians about what

Platonism precisely entails. Given the diverse types of Platonism, for this study, "Platonism" will refer to Gödelian Platonism, chosen for its strong assertion of a non-sensual mathematical reality, where mathematical entities exist independently of human perception or physical senses in a timeless, abstract realm, and its influence on modern set theory, as articulated by Gödel.

Platonism is significant, as roughly 80% of mathematicians identify with it (Abbott, 2013). However, it faces substantial challenges from physicalist and naturalist theories of knowledge, which argue that all knowledge must arise from physical processes or empirical interactions with the world. These theories object to Platonism's positing of abstract, non-spatiotemporal mathematical entities, as it is unclear how such entities, devoid of physical or causal presence, can be known by human minds rooted in the physical world. This tension leads directly to Benacerraf's epistemological problem, which questions how humans, as spatiotemporal beings, can access or gain reliable knowledge of such abstract mathematical objects (Benacerraf, 1973). Even alternative epistemologies like reliabilism, which focus on the reliability of belief-forming processes, struggle to justify mathematical knowledge under Platonism, as they cannot easily account for a causal connection to non-physical entities.

Davies (2007) argues that mathematical intuition is a neuropsychological illusion, arising from prolonged engagement with mathematical concepts rather than access to a Platonic realm of objective truths. Functional Magnetic Resonance Imaging (fMRI) studies, he claims, demonstrate that the brain constructs mathematical realities through neural processes, not by perceiving an independent mathematical world. This view is in parallel with the historical misperception that Euclidean geometry was an absolute, universal truth about physical space, a belief that later challenged by non-Euclidean geometries, which revealed alternative frameworks could equally (or better) describe reality.

Then Hersh (2008) further critiques Platonism by arguing mathematics is culturally conditioned. He compares Platonist objectivity claims to past mistaken beliefs about natural social orders, suggesting that mathematics appears objective from within a cultural framework but has no independent existence.

Nonetheless, Platonism finds strong support in mathematics' effectiveness in science. Wigner (1995) famously called this "the unreasonable effectiveness of mathematics." The Indispensability Argument, advanced by Quine and Putnam, contends we must accept the existence of mathematical entities because they are indispensable to our best scientific theories. Colyvan (2024) summarizes the argument:

> Premise 1: We must commit to entities indispensable to our best theories.
>
> Premise 2: Mathematical entities are indispensable.
>
> Conclusion: We are committed to mathematical entities.

Putnam (1979) claimed that rejecting mathematical entities while accepting scientific realism is akin to believing in angels without believing in God, as both mathematical and physical entities are justified by their indispensability to our best scientific theories, which are empirically confirmed as a whole. However, critics like Field and Maddy (1992) challenge the validity of indispensability premises. For example, they attack the second premise, arguing that mathematical entities are not truly indispensable, as scientific theories like Newtonian physics can potentially be reformulated without reference to abstract objects by using nominalist strategies. Additionally, Cheyne & Pigden (1996) argue that the indispensability of mathematical entities in scientific theories (such as in General Relativity or quantum mechanics) implies they must have causal efficacy, which poses a challenge for Platonism's view of abstract, acausal objects of mathematics. This contradicts Platonism's assertion that mathematical entities exist independently of causal interactions. Thus, if mathematical objects are indispensable, Platonism struggles to explain their lack of causal influence.

David Mumford (2008) provides further support for Platonism by noting mathematics' apparent universality and independence of culture, suggesting that mathematical truths hold across all human societies. Yet, Øystein (2024) notes that this universality may reflect truth-value realism rather than a commitment to ontological mathematical objects, as truth-value realism holds that mathematical statements, like 'there are prime numbers between 10 and 20,' have objective truth-values independent of whether they are known or derivable, without requiring the existence of abstract entities like numbers. This view, endorsed by some nominalists, posits that such truths can be explained through alternative translations into a philosophical language that avoids reference to mathematical objects, thus aligning with Mumford's observation of universality without necessitating Platonism's ontological claims.

While Platonism remains the dominant philosophical stance among mathematicians, its foundations face serious challenges. Platonists must overcome significant epistemological objections, such as Benacerraf's (1973) problem of how humans, as spatiotemporal beings, can access abstract mathematical objects lacking physical presence. Additionally, metaphysical objections arise, notably from Cheyne & Pigden (1996), who argue that the indispensability of mathematical entities in science implies causal efficacy, conflicting with Platonism's claim that such entities are acausal and exist independently of the physical world.

2. Formalism in the Philosophy of Mathematics

Let us now turn to the first alternative of Platonism, namely Formalism, which is a branch of Nominalism. In contrast to Platonism, Formalism posits that mathematics is merely the formal manipulation of symbols according to a set of rules known as axioms (Weir, 2025). Mathematics, in this view, resembles a game like chess: if we change the rules, we are simply playing a different game (Weir, 2025). A typical illustration is Euclidean geometry, based on five axioms, one of which states that given a line and a point not on that line, there is exactly one line through the point parallel to the original. For centuries, the truth of these axioms was considered absolute; Kant even believed Euclidean geometry to be psychologically inevitable, arising naturally from our spatial intuition (Franklin, 2014a).

However, in the early 19th century, Bolyai and Lobachevsky independently demonstrated that one can abandon the parallel postulate and develop a new form of geometry. This non-Euclidean geometry provides an equally coherent and useful description of space. This discovery undermined the notion that the axioms of Euclidean geometry were uniquely determined by human intuition.

According to the formalist perspective, mathematical objects, relations, and structures do not truly exist (Weir, 2025). The truths expressed in logic and mathematics are not about anything; they are purely formal and thus, in a sense, meaningless. Mathematical statements are viewed as syntactic manipulations, devoid of reference to any external reality.

David Hilbert, an early proponent of Formalism, sought to establish a program that would achieve a consistent and complete axiomatization of all mathematics. He envisioned a formal system in which any mathematical statement could be proven true or false solely through the manipulation of axioms within a formalized language (Weir, 2025). However, this ambitious goal was later shattered by Kurt Gödel’s incompleteness theorems, which showed that within any consistent formal system capable of expressing basic arithmetic, there exist true statements that cannot be proven within the system. These results demonstrate that any sufficiently powerful formal system cannot be both complete and consistent. As a result, Hilbert's dream of fully formalizing mathematics was proven unattainable.

Since Hilbert's time, Formalism has evolved. Most contemporary formalists no longer adhere to Hilbert's strict vision. Instead, they entertain the possibility that computer algorithms might eventually automate the proof process, taking over the human role in verifying mathematical proofs (Weir, 2025).

Nevertheless, Formalism faces a major philosophical challenge: it struggles to account for the remarkable applicability of mathematics to the real world. If mathematics is merely a meaningless game played according to arbitrary rules, there seems to be no reason why it should be so uniquely effective in helping scientists make discoveries about the natural world. If we could simply discover some different axioms and different rules, as Formalism suggests, why is it that only mathematics, this particular "game", proves to be indispensable to scientific progress? This question arises because Formalism views mathematical truths as derived from human constructed systems rather than inherent properties of reality, yet no other symbolic system has demonstrated the same utility in explaining natural phenomena.

To date, there is no alternative symbolic game that rivals mathematics in its utility for science. Mathematics appears to be essential to our understanding of physical reality. Hence, the analogy between mathematics and games, often invoked by formalists, seems fundamentally flawed. Mathematics is not merely a game; there is no other "game" quite like it.

3. Logicism in the Philosophy of Mathematics

Another branch of Nominalism is Logicism, which was influential until around 1930, after the impact of Gödel's incompleteness theorems and the rise of Zermelo-Fraenkel set theory (Tennant, 2023). Even though logicism is a form of Nominalism, it differs from Formalism.

Traditionally, Logicism has focused on arithmetic and real analysis, while Formalists, notably David Hilbert, left integers undefined, treating them merely as symbols within a formal system governed by axioms and rules, without requiring an intrinsic meaning or reference to external entities (Russell, 1903). For a Formalist, numbers like 0, 1, and 2 have no meaning beyond the axioms, hence this approach prioritizes the formal structure of mathematics over any semantic interpretation of numbers as objects. In contrast, Logicists aimed to show that mathematics is reducible to logic. Prior to the rise of Logicism, logic was regarded mainly as a philosophical tool. Logicism shifted this view by asserting that mathematics and logic are identical (Russell, 1903). Accepting Kant's distinction between analytic and synthetic truths, Logicism treated mathematical statements as analytic truths within a logical framework. In Principia Mathematica, Russell and Whitehead aimed to reduce mathematics to logic by deriving mathematical truths from logical axioms, as seen in their development of type theory to define numbers logically. While the work's mathematical rigor might suggest a reverse dependency, their intent was to establish logic as the foundation, not to reduce logic to mathematics, aligning with the core Logicism thesis.

Two major versions of Logicism emerged: Fregean and Russellian. Influenced by Dedekind's reduction of real numbers to rationals using set theory, Frege aimed to reduce arithmetic to logic without appealing to psychological intuition. Frege conceived numbers as objects, adopting a Platonist stance. His system, articulated in Grundgesetze der Arithmetik, sought to derive arithmetic from logical axioms, treating truths like 2 + 5 = 7 as being analytic (Tennant, 2023).

Frege defines numbers using logical constructions: zero as the number of objects not identical to themselves, one based on zero, and so forth (Franklin, 2014a). However, his project depended on Basic Law V, claiming that two properties are identical if they apply to the same objects, a principle intended to define sets logically. This law led to Russell's Paradox, which exposed an inconsistency in Frege's system by considering a set of all sets that do not contain themselves: if this set contains itself, it must not contain itself, and if it does not contain itself, it must contain itself, creating a logical contradiction (Russell, 1903). This paradox revealed that Frege's logical foundation for arithmetic was inherently self-contradictory, undermining his logicist aspiration. Profoundly disheartened, Frege abandoned the project, lamenting in Grundgesetze Volume II: “Hardly anything more unwelcome can befall a scientific writer than to have one of the foundations of his edifice shaken after the work is finished” (Frege, 2013: 253).

Despite recognizing the paradox, Russell pursued Logicism in a revised form. Russell's Logicism became less Platonist and subsequently admitted: “When I wrote the ‘Principles’, I shared with Frege a belief in Platonic reality of numbers… a comforting faith, which I later abandoned with regret” (Russell, 1903: x). For Russell, logic concerned ways of constructing statements, not the study of objectively existing forms. He rephrased mathematical claims such as "There are two cats" into logical expressions by using relational terms instead of explicitly invoking numbers. For example, he might express it as a statement about the relationship between two distinct objects, ensuring the focus remained on logical structure rather than numerical quantities. Thus, mathematical propositions were seen as tautologies, analytic truths by definition. Nonetheless, Russellian Logicism faced challenges. Logical elimination of set-theoretic notions like "membership" proved impossible, and the Axiom of Infinity, necessary for set theory, seemed non-logical (Franklin, 2014a).

Attempts to revive Logicism emerged as Neo-Logicism in the mid-20th century. Crispin Wright (1983) proposed Hume’s Principle, that the number of F’s equals the number of G’s if and only if they can be paired one-to-one, as a foundation for arithmetic. Though not vulnerable to Russell’s Paradox, it remains unclear whether Hume’s Principle is a logical or analytic truth.

While Nominalism has an epistemological edge over Platonism, it faces other serious problems. It struggles to explain why mathematics is so successful in scientific theorizing. If mathematical objects do not exist, how does reference to them contribute to empirical success? Formalists and Logicists also fail to fully account for mathematics’ deep objectivity. As Franklin (2014b) notes, mathematical inquiry uncovers a pre-existing structure that seems independent of human invention.

Thus, while Platonism and Nominalism both capture important aspects of mathematical practice, neither provides a fully satisfactory account. The persistent limitations of both views suggest the need for a new direction in the philosophy of mathematics.

4. Aristotelian Realism Philosophy of Mathematics

Aristotle, a realist and student of Plato, adapted his teacher's concept of universals, such as redness and oneness, by grounding them in physical objects rather than locating them in a transcendental realm of forms. For Aristotle, the geometrical shape of a vase, for instance, exists inherently within the vase itself, not in a separate abstract world (Gardner, 2009). In contrast to Plato, Aristotle maintained that the properties of objects are real and exist within the objects themselves. The philosophy of mathematics that embraces this view is known as Aristotelian realism.

Unlike Platonism and Nominalism, which often focus on abstract entities like sets, numbers, and logic, Aristotelian realism is rooted directly in applied mathematics (Franklin, 2014b). It views mathematics as a science about the real world, not about a separate transcendental realm (as in Platonism) nor as a meaningless manipulation of symbols (as in Formalism) (Lear, 1982). According to this view, mathematical properties such as symmetry, continuity, and order are realized in the physical world. Thus, mathematics describes real aspects of reality just as biology or physics does (Franklin, 2014a). On this account, the real world is understood as a state of affairs in which particulars instantiate universals, and universals are genuine parts of the physical world.

One version of Aristotelian realism discussed here is modal Aristotelianism (or semi-Platonism), which holds that "universals can exist and be perceived in this world and often do, but it is a contingent matter which ones exist, and we can have knowledge even of uninstantiated universals and their necessary interrelations" (Franklin, 2014a: 26). Another version, strict this-worldly Aristotelianism, will not be discussed here.

Our senses and our capacity for reasoning enable us to grasp mathematical facts. For instance, we understand that $2 \times 3 = 3 \times 2$ by arranging six objects into either two rows of three or three columns of two. Such relations can be directly realized and observed without invoking otherworldly entities. Similarly, mathematical proofs involve intellectual insights that go beyond mere perception, binding multiple insights into a coherent structure that reveals necessary truths. Many mathematical necessities can be realized in the world, as readily as sensory properties like color or shape. Comparative judgments, such as discerning that one object is taller than another, demonstrate that relations like "being taller than" are perceptible and repeatable.

In Aristotelian realism, universals do not exist in an abstract realm but are embedded within physical objects and possess causal power. For example, objects emit signals, such as light, that interact with our senses precisely because of their properties like shape or color. We recognize a square object because its shape distinctively affects our visual perception, just as its color would. Structural features such as symmetry and order are directly observable and causally efficacious. This gives Aristotelian realism a significant epistemological advantage over Platonism, which struggles to explain how we have access to causally inert abstract entities. Aristotelian epistemology, by contrast, accounts for both sensory and intellectual knowledge, though a full treatment of this distinction is beyond our scope here.

A common objection to Aristotelian realism concerns uninstantiated universals, such as an unseen shade of blue or numbers larger than the number of particles in the universe. Modal Aristotelianism addresses this by positing that uninstantiated properties belong to structured ranges of universals, known as determinables (Franklin, 2014a). For instance, an unexperienced shade of blue must lie between two shades we have experienced. Since our senses respond continuously to variations in color and length, we can infer the existence of intermediary properties even without direct experience. Similarly, facts about betweenness and ratios, such as the necessary relations between different lengths, seem logically necessary and are not contingent on instantiation. For example, there is no possible world in which, if A is twice as long as B and B is twice as long as C, then A is not three times as long as C.

As Nominalism and Platonism continue to challenge each other by exposing mutual shortcomings without resolving their own foundational problems, Aristotelian realism emerges as a promising alternative. It offers a fresh direction in the philosophy of mathematics. Although still developing, Aristotelian realism's epistemology appears far more robust than Platonic epistemology. By recognizing the existence of universals while situating them within the real world, Aristotelian realism strikes a compelling middle ground between Platonism and Nominalism. There are further reasons to favor Aristotelian realism beyond those discussed here, suggesting it may offer the most coherent framework for understanding the nature of mathematics.

5. Applicability of Mathematics

Dominant schools of thought, such as Platonism and Nominalism, focus primarily on the ontological status of mathematical objects like sets and numbers, often neglecting the practical question of why mathematics is so successful in real-world applications. This oversight manifests in several ways: traditional philosophies tend to prioritize abstract debates about the existence or nature of mathematical entities over empirical investigations into how mathematics effectively models physical phenomena, such as predicting planetary motion or explaining quantum mechanics. Moreover, these schools rarely address the historical success of mathematical tools in scientific discovery, leaving the "unreasonable effectiveness" highlighted by Wigner (1995) largely unexamined in their frameworks.

Platonism posits that mathematical entities, such as numbers and geometric forms, exist in a non-spatiotemporal, abstract realm, independent of the physical world and human minds. This view, rooted in the philosophy of Plato, suggests that mathematical objects have no causal interaction with the empirical world, which creates a significant challenge: how can such abstract entities be so relevant to describing physical phenomena? Nominalism, such as Formalism and Logicism, takes a contrasting stance by denying the existence of mathematical objects altogether (Bueno, 2013). Nominalists argue that mathematical terms are mere linguistic constructs or useful fictions, devoid of ontological reality. This position, however, struggles to account for the empirical success of mathematics. If mathematical entities do not exist, how can referencing them in scientific theories lead to accurate predictions and explanations? For example, how can nominalists explain the indispensable role of mathematical structures in disciplines like physics, where equations reliably describe natural phenomena? Both Platonism and Nominalism, by focusing on the nature of mathematical objects, fail to provide satisfactory accounts of applied mathematics, highlighting the need for an alternative philosophical perspective.

Steiner (1998) directly addresses this gap by examining why mathematics is so effective in the physical sciences and exploring the implications for our understanding of the universe and the human mind's place within it. Steiner's work is divided into several key discussions, each tackling different facets of mathematical applicability. In the first chapter, "The Semantic Applicability of Mathematics: Frege's Achievements," he gives some credit to Gottlob Frege for resolving critical semantic and metaphysical issues related to the applicability of mathematics. Frege argued that mathematical terms, such as the number "twelve" in the statement "There are twelve fruits on the table," refer to abstract numbers that characterize empirical concepts rather than physical objects. For instance, the number twelve does not directly describe the fruits but quantifies the concept of the collection of fruits (Steiner, 1998: 16). This distinction allows mathematics to bridge the gap between abstract entities and real-world applications by applying to empirical concepts used in scientific descriptions. Frege's solution, Steiner argues, extends to all mathematical terms, providing a semantic framework for understanding how mathematics connects to the physical world.

Steiner then poses a broader question: how do mathematical concepts enable us to describe and predict physical phenomena? In addressing this, he begins with simple mathematical operations like addition and multiplication. To illustrate this, he introduces the concept of dispersion, defined as the average distance between any two elements in a set. He suggests that sets with low dispersion are more "interesting" to humans, as they represent objects that are physically closer together, such as a cluster of items (Steiner, 1998: 26). When sets are combined, he claims, their dispersion typically increases, especially if the elements are spatially separated. However, Simons (2001) challenges this assertion with a counterexample involving the vertices of a Star of David inscribed in a circle of radius 1. The two sets of vertices from the equilateral triangles each have a dispersion of approximately 1.732 (the square root of 3), while all six vertices together have a dispersion of about 1.493. This shows that dispersion can decrease when sets are interspersed, undermining Steiner's generalization and highlighting the complexity of applying mathematical concepts to physical systems.

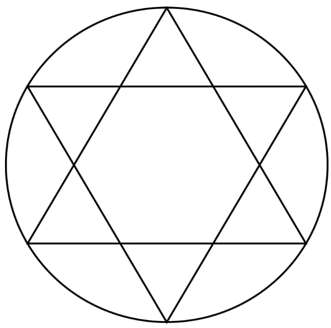

Fig. 1. Star of David, formed by two overlapping equilateral triangles, inscribed in a circle of radius 1.

How specific mathematical concepts are suited to describing natural phenomena is also discussed in Steiner's work. He argues that the applicability of each concept must be addressed case by case, rather than seeking a universal explanation (Steiner, 1998: 47).

For example, the concept of linearity is particularly effective in systems where the Principle of Superposition holds, such as in wave mechanics. Steiner frames this discussion within Wigner's quandary about the "unreasonable effectiveness of mathematics," viewing it as an epistemological puzzle concerning the relationship between the human mind and the cosmos. While Steiner identifies this as a profound issue, he does not provide a conclusive resolution, suggesting that the problem requires further philosophical exploration.

Detailed accounts of how physicists have leveraged mathematical analogies to achieve groundbreaking discoveries are also provided. These analogies, often Pythagorean in nature, are expressed solely in the language of mathematics, transcending verbal or physical descriptions. A subset of these, which Steiner terms "formalist," involves manipulating mathematical symbols without assigning immediate physical meaning. A striking example is Maxwell's modification of Ampere's law, where he introduced the concept of displacement current, a rate of change of an electric field, distinct from conduction current. This led to the formulation of Maxwell's equation:

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

where E is the electric field, B is the magnetic field, J is the current density, μ_0 is the permeability of free space, and ϵ_0 is the permittivity of free space. This equation unified electromagnetism, revealing that a changing magnetic field induces a changing electric field, and vice versa, resulting in electromagnetic waves that propagate at the speed of light (Colyvan, 2001b). Maxwell's discovery, driven by mathematical analogy rather than empirical observation, not only predicted electromagnetic phenomena but also confirmed that light itself is an electromagnetic wave. This exemplifies the predictive power of mathematics, as the implications of Maxwell’s equations extended far beyond their initial assumptions.

Steiner (1998: 75) argues that such successes challenge naturalism, which is the view that scientific inquiry should rely solely on empirical observation and avoid anthropocentric assumptions. Instead, he advocates an anthropocentric perspective, suggesting that nature is “user-friendly” to human goals and values, aligning with human cognitive capacities in a way that facilitates discovery. This view implies that humans occupy a privileged position in the universe, a stance that borders on theistic interpretations. It is worth noting that his assertion of a "user-friendly" universe where nature aligns with human cognitive capacities lacks empirical grounding and relies heavily on unproven assumptions. This perspective suggests that the effectiveness of mathematical analogies, such as Maxwell’s equations, stems from a privileged human position in the cosmos, yet it struggles to explain the mechanism behind this alignment. Furthermore, as Simons (2001) critiques, this stance contradicts the anti-anthropocentric tradition of scientists like Copernicus, Galileo, Newton, and Darwin, who decentred humanity in their cosmological models. Steiner’s focus on successful cases of mathematical applicability also introduces a bias that overlooks instances where mathematics fails to fit, while his claim that scientists have abandoned naturalism lacks robust evidence, leaving his anthropocentric thesis vulnerable to scepticism.

As opposed to Steiner’s anthropocentric view that posits a contingent alignment between human cognition and nature, Aristotelian realism asserts a necessary connection grounded in the physical instantiation of mathematical properties. For example, the ratio of lengths in a physical object pre-exists measurement, which merely identifies and quantifies these ratios, with units added for computational convenience. This perspective grounds mathematics in the empirical world, making its applicability less mysterious.

Another perspective on mathematical applicability emerges from the standard view, which posits that informal proofs reliably indicate the existence of formal derivations within axiomatic systems (Avigad, 2021). This reliability is enhanced through strategies such as generalization for strengthening lemmas to expose errors, and modularity for breaking proofs into manageable components. These practices, as seen in formal verifications like the Kepler Conjecture, suggest that mathematics' effectiveness stems from its ability to mirror real-world structures.

Franklin criticizes traditional philosophy of mathematics for its "intellectual anorexia," focusing narrowly on numbers and set theory while neglecting applied mathematics (Franklin, 2014a). Aristotelian realism, by contrast, embraces fields like geometry, group theory, combinatorics, and formal sciences such as operations research and information theory, which study structural properties directly relevant to the physical world. For instance, the measurement of lengths reveals ratios that exist independently of human intervention, and mathematical models of these ratios can predict physical behaviors. This approach answers Wigner's question about the unreasonable effectiveness of mathematics by asserting that mathematics is effective because it studies properties intrinsic to the physical world. By focusing on applied mathematics, Aristotelian realism not only overcomes the limitations of Platonism and Nominalism but also remedies the philosophical neglect of this critical field.

6. Mathematical Modelling and Simulations

Mathematics' extraordinary capacity to describe, measure, and evaluate phenomena is one of its most powerful features. As Galileo Galilei declared in The Assayer (1960), the universe is written in the language of mathematics, with its characters being triangles, circles, and other geometric figures, without which understanding the world is impossible. From mundane activities like saving money or throwing a baseball to complex events like towing icebergs or meteors colliding with Earth, mathematics provides precise, concise descriptions through mathematical models.

A mathematical model is a representation of a system using mathematical concepts and language (Dym, 2004). Effective models explain systems, analyse components, and predict outcomes. The process of creating such models, known as mathematical modelling, is fundamental to natural sciences, particularly physics, and engineering disciplines like computer science. It also plays a central role in formal sciences, such as operations research, which emerged during World War II to optimize military strategies, such as determining the most effective search patterns for U-boats or the optimal size of convoys. Despite its significance, philosophers have paid scant attention to formal sciences, which achieve rigorous proofs in areas like network flows and program correctness, resembling a "philosopher's stone" that transforms contingent observations into necessary knowledge (Franklin, 2014a).

Mathematical models typically consist of variables and equations that establish relationships between them. By providing a structured representation of these relationships, this approach lays the groundwork for the scientific method, which involves three stages: observation, modelling, and prediction (Dym, 2004). In observation, empirical data is collected, either directly through senses or indirectly through measurements, such as observing the products of a chemical reaction. Modelling analyses these observations to describe behaviours, explain causes, or predict future outcomes. Prediction tests models' validity by comparing their outputs to real-world events. Models enable surrogate reasoning, allowing scientists to study systems indirectly (Frigg & Hartmann, 2025). For reliability, models must be structurally stable, meaning small changes in inputs do not drastically alter predictions.

Consider the equation $Pt = P_{t-1} + (1/100)P_{t-1}$, which models compound interest at 1% per month, where P_{t-1} is the amount at month $t-1$. This equation describes a local structure, the relationship between successive months and yields a global structure of exponential growth after iterative calculations. Remarkably, the same model applies to the temperature of an iron rod, where P_t represents the temperature at a point t notches from the left, increasing by 1% per notch. This versatility across domains (money, temperature) and dimensions (time, space) underscores mathematics' focus on structure, supporting Aristotelian realism's view that mathematics is the science of quantity and structure.

Many mathematical models are idealizations, simplifications like frictionless planes or perfect spheres, that may create gaps between model predictions and reality. Platonism views these as reflections of an abstract mathematical realm, but Aristotelian realism challenges this. Franklin (2014a) demonstrates this using the Königsberg bridge problem, where the impossibility of crossing all bridges without retracing one is a structural property of the actual system, not an idealized version. The proof depends on connectivity, not distances or areas, which eliminates any idealization-reality gap. Aristotelian realism replaces idealization with approximation, treating complex systems as closely resembling simpler structures. For example, an imperfect bronze sphere's volume can be approximated using a perfect sphere's formula, as their boundaries are nearly identical. This approach avoids Platonism's abstract-physical divide, grounding mathematics in empirical reality.

Models vary ontologically: material (e.g., wooden ship models), fictional (e.g., frictionless pendulums), set-theoretic structures, descriptions, equations, or combinations (Frigg & Hartmann, 2025). Material models, like Watson and Crick's DNA model, are physical and straightforward, raising only metaphysical questions about objects. Fictional models, existing only in the mind, face ontological challenges, with critics like Quine (1953) arguing that fictional entities lack existence and should be reframed as predicates. Set-theoretic models, prevalent in mathematized sciences, are criticized for not explaining construction or representation without additional assumptions (Cartwright, 1999). Models as descriptions or equations, such as the Black-Scholes model, encounter issues because different descriptions or coordinates may represent the same model, suggesting distinct properties.

Modelling often follows a cycle: formulate, develop, validate, and reformulate. For example, the Malthusian equation for population growth, $\frac{dN}{dt} = aN$, yields exponential growth $N(t) = N_0 e^{at}$, but ignores growth limits, making it unrealistic. The Verhulst equation,

$$\frac{dN}{dt} = aN\left(1 - \frac{N}{N^*}\right),$$

with solution

$$N = \frac{N^* N_0}{N_0 + (N^* - N_0)e^{-at}},$$

caps growth at the carrying capacity N^*, better reflecting reality (R. Banks, 2013).

From the perspective of Aristotelian realism, this modelling cycle exemplifies how mathematics captures real-world properties, such as quantity and structure, inherent in physical systems. In the case of population growth, the Malthusian model initially approximates the quantitative property of population increase but fails to account for the structural constraint of a carrying capacity. The reformulation into the Verhulst model aligns more closely with reality by incorporating this limit, reflecting the actual behaviour of population growth. This iterative process of refining models to better represent empirical structures supports Aristotelian realism's claim that mathematics is grounded in the physical world, not an abstract realm as Platonism suggests.

Mathematical models also often rely on advanced probabilistic frameworks to handle infinite or complex systems, as seen in the development of non-Archimedean probability functions that use ultrafilters to address challenges like the fair lottery paradox on the natural numbers (Horsten & Brickhill, 2024). This approach, detailed in their analysis of generalized infinite additivity, enhances the applicability of mathematics in simulations by providing a robust method to model scenarios such as random dart throws on the real unit interval.

The cycle of formulation, validation, and reformulation mirrors the Aristotelian approach of studying real properties through measurement and approximation, ensuring that mathematical models remain tethered to observable phenomena. Mathematical models often rely on equations that may yield closed-form analytical solutions. However, when analytical solutions are infeasible, computer simulations become essential. Simulations, widely used in natural and social sciences, model complex phenomena such as galaxy formation, high-energy ion reactions, evolutionary processes, economic trends, and conflict dynamics (Winsberg, 2022).

J. Banks et al. (2001) define a simulation as “an imitation of a real-world process or system,” typically applied to dynamic models involving time. The goal is to solve equations that represent a system’s temporal evolution, with one process mimicking another. Simulations have sparked debate about their philosophical implications. Some argue they represent a new scientific paradigm, raising novel epistemological questions, while others contend that they pose few unique philosophical challenges (Frigg & Hartmann, 2025). Regardless, their practical importance is undeniable, enabling exploration of dynamic models when traditional methods fail, thus “extending” human analytical capabilities (Humphreys, 2004).

A critical issue is the reliability of simulation results, which hinges on two questions: do the model’s equations accurately describe the target system (validation), and does the computer provide sufficiently precise solutions (verification)? These are difficult to assess independently, as only the simulation’s “net outcome” is observable (Frigg & Hartmann, 2025). Equation-based simulations, prevalent in physics and related fields, are guided by governing theories and differential equations. They are classified as particle-based, modelling interactions among discrete bodies (e.g., galaxy formation via gravitational forces), or field-based, addressing continuous media (e.g., storm dynamics, where fluid variables are discretized in space and time) (Winsberg, 2022).

Simulations serve multiple purposes: heuristic exploration, predicting unavailable data, understanding existing data, and anticipating system behaviour under specific conditions. They can forecast future events, reconstruct past phenomena, or explain observed behaviors by clarifying underlying mechanisms. Simulations often focus on structural properties, allowing flexibility in application. For instance, a model describing accumulated money after t months, $P_t = P_{t-1} + (1/100)P_{t-1}$ above, can also represent the temperature along an iron rod, with P_t as the temperature at t notches from the left. This versatility highlights that simulations model structural relationships, independent of whether the variable represents money, temperature, time, or space.

Aristotelian realism offers a compelling lens for understanding simulations. It asserts that the structures modelled in simulations, such as growth patterns or physical interactions, are inherent in the physical world. In the example of the money-temperature model, the simulation captures a structural property (exponential change) that exists in both financial and thermal systems. This aligns with Aristotelian realism's emphasis on mathematics as a study of empirical properties, not idealized abstractions.

Simulations, by approximating these real structures through discretized equations, reflect the iterative process of measuring and refining models to match observable phenomena. The ability to apply the same model across domains underscores the universality of structural properties in nature, which supports the idea that mathematics is grounded in physical reality. This perspective enhances our understanding of simulations as tools that reveal the intrinsic mathematical order of the world, bridging theoretical models and empirical outcomes.

By positing that mathematical properties are inherent in the physical world and thus accessible to both human observation and AI analysis, Aristotelian realism gains relevance. For instance, AI-driven simulations of physical systems such as climate modelling or quantum computing optimizations rely on mathematical structures abstracted from real-world data, aligning with this view. Unlike Steiner's anthropocentric perspective, which attributes AI's success to a contingent human-nature alignment, Aristotelian realism offers a necessary connection, explaining why mathematics underpins AI's empirical success. The novelty lies in this synthesis: the debate over AI's mathematical capabilities challenges traditional philosophies to adapt, positioning Aristotelian realism as a dynamic framework that not only accounts for historical applicability but also anticipates future technological integration. This shift prompts a rethinking of mathematics' nature, moving beyond abstract speculation to a philosophy grounded in its active role in shaping modern science and technology.

CONCLUSION

A comprehensive philosophy of mathematics must confront the central challenge of explaining the applicability of mathematics to the physical world, a theme that has been a recurring focus throughout this article. Platonism, with its assertion of mathematical objects in a non-spatiotemporal realm, fails to bridge the gap to empirical reality. Similarly, Formalism and Logicism, by framing mathematics as either a rule-based game or a logical construct, have failed in accounting for its profound influence on scientific discovery.

Steiner highlights this issue, arguing that the success of mathematical analogies, such as Maxwell's equations derived through Pythagorean reasoning, reveals an "unreasonable effectiveness" that challenges naturalism and suggests an anthropocentric, "user-friendly" universe. However, Steiner's anthropocentrism lacks a fully satisfying explanation for this effectiveness, as it rests on the unproven assumption that the universe is inherently structured to align with human cognitive capacities. This view struggles to provide a robust mechanism for why such alignment occurs, leaving the effectiveness of mathematics as an unexplained coincidence rather than a necessary feature of reality.

Aristotelian realism, in contrast, offers a compelling solution, viewing mathematics as the science of real-world properties like structure and quantity. This perspective grounds applied mathematics in physical reality, resolving the impasse between Platonism and Nominalism. By positing that mathematical properties are inherent in the physical world and accessible through observation and abstraction, Aristotelian realism provides a coherent explanation for mathematics' applicability. For its ability to explain the applicability of mathematics through empirically grounded principles, Aristotelian realism emerges as a superior framework and a significant breakthrough in the philosophy of mathematics.

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